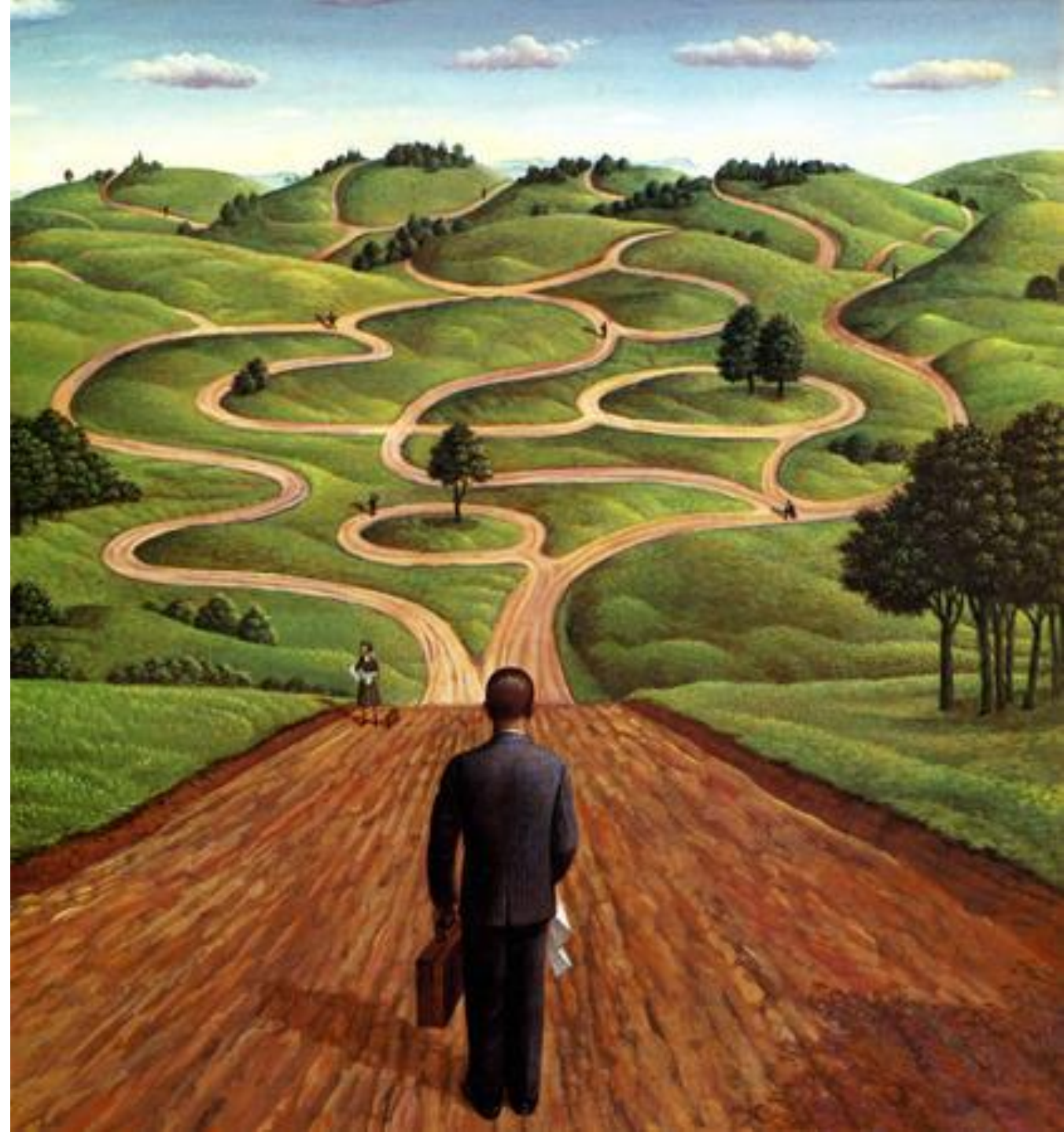


# Paths through morphospace

Morphometrics, phylogenetics, and evolution

P. David Polly  
Indiana University  
[pdpolly@pollylab.org](mailto:pdpolly@pollylab.org)  
<https://pollylab.org/>



# Previous Rohlf Medalists



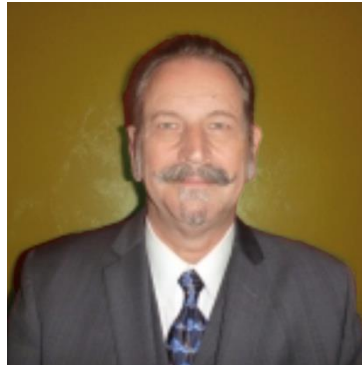
Bookstein



O'Higgins



Hallgrimsson



Slice



Adams

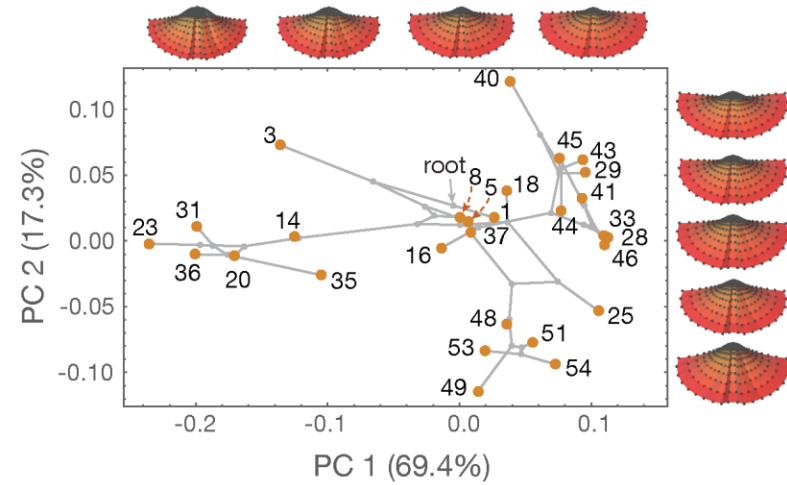
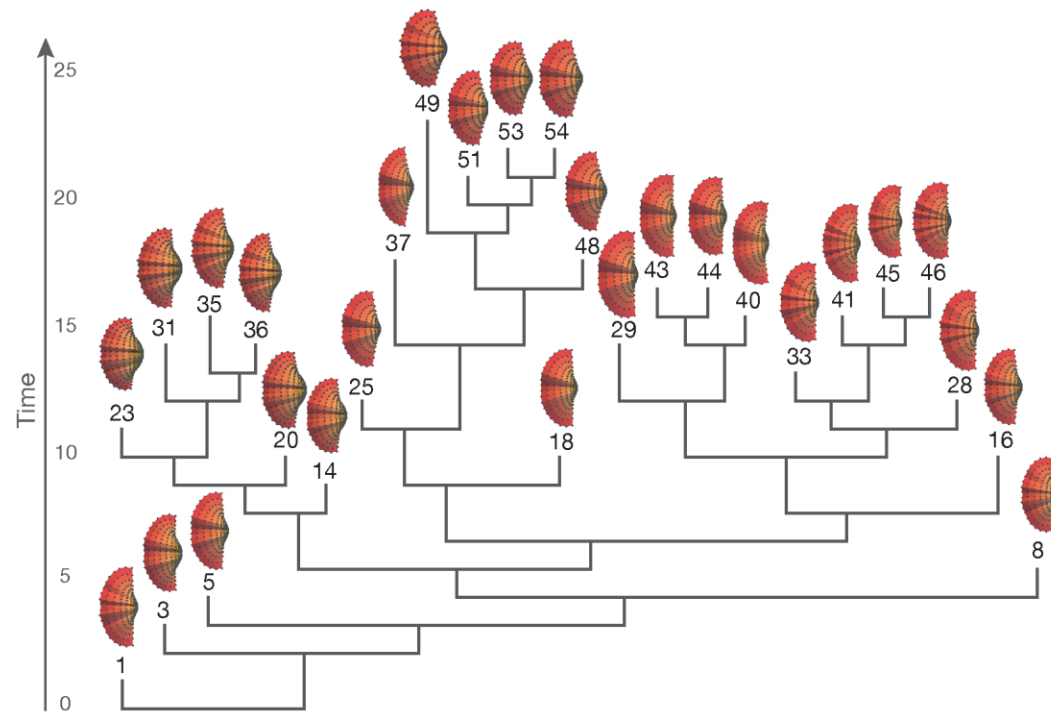


Richtsmeier



Mitteröcker

# Phylogeny and morphometrics



Polly PD & Motz GJ, 2017, *Paleontological Society Papers*, 22: 71-99.



Paths through morphospace

# Is it always possible to estimate...



Thomas Hart Benton, *High Plains* (1953)

- evolutionary transitions?
- ancestor shape reconstructions?
- phenotypic change on a phylogeny?
- rates or modes of evolution?
- adaptive landscapes?

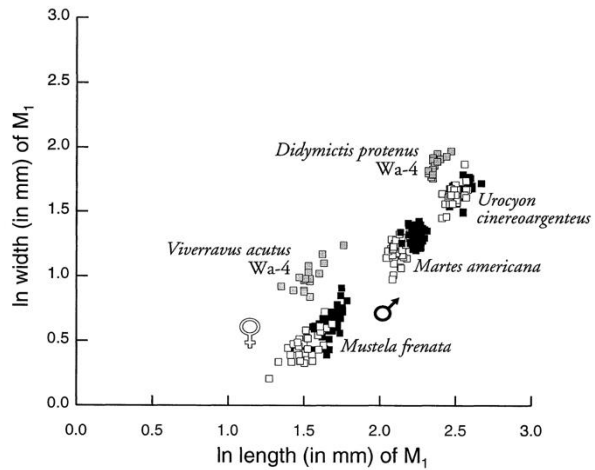
In what ways do evolutionary paths  
map onto mathematical paths?



## Paths through morphospace

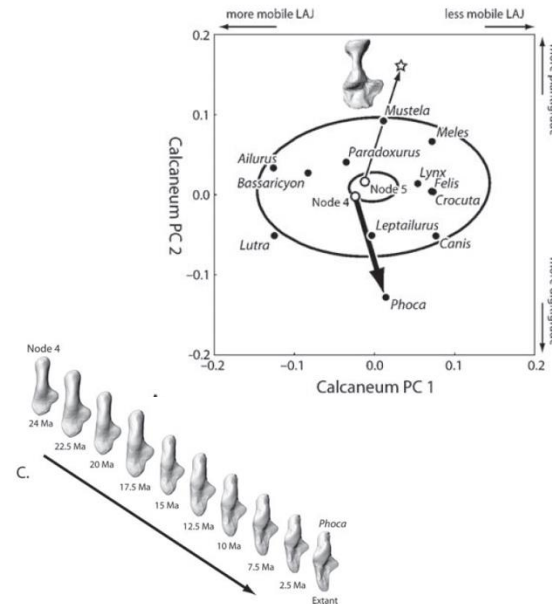
# We estimate paths in nearly every morphometric study

### Taxonomic differences



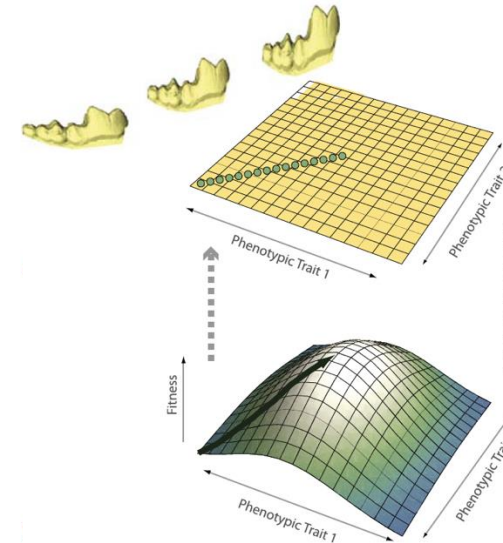
Polly PD, 1998. *Contr. Mus. Paleo. U. Mich.*, 30: 1-53.

### Evolutionary trajectories



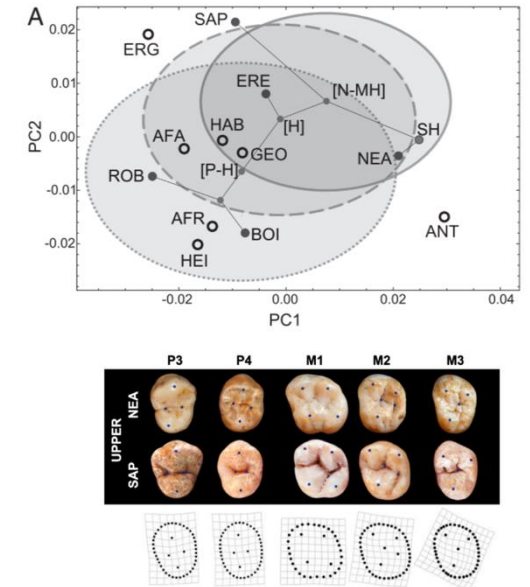
Polly PD, 2008. *Mammalian Evolutionary Morphology*, 167-198.

### Modeling and simulation



Polly PD, 2008. *Evol. Biol.*, 35: 85-96.

### Ancestor reconstruction

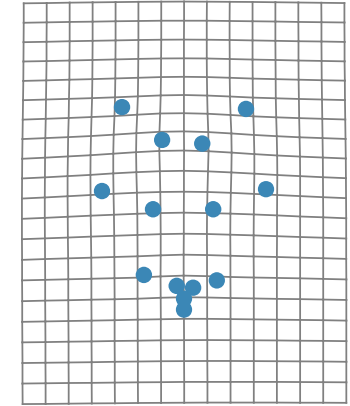
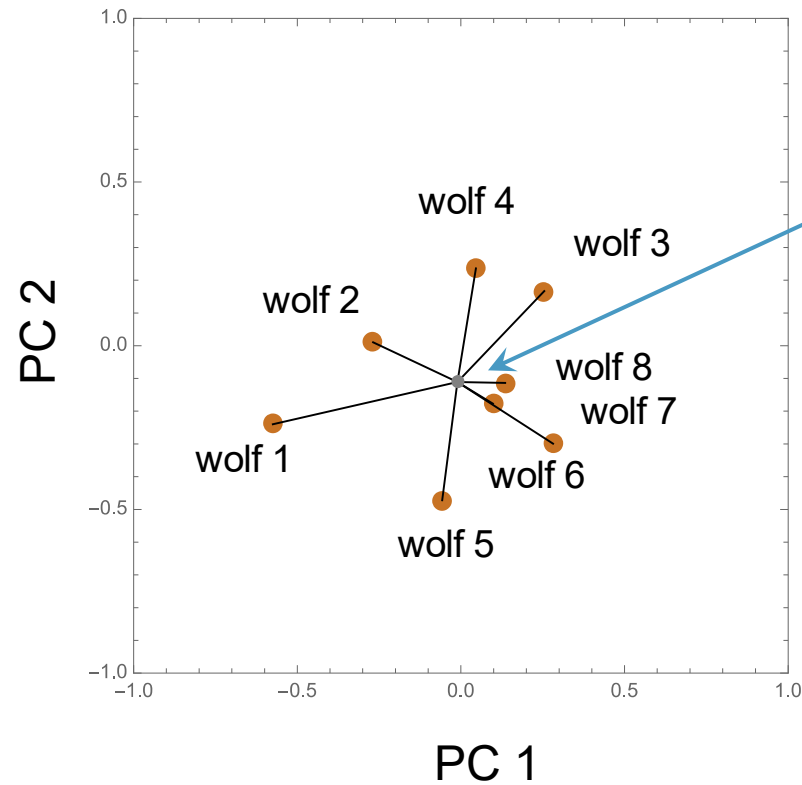


Gomez-Robles et al. 2013. *PNAS*, 110: 18196-18201.



Paths through morphospace

Even a population means involves paths...

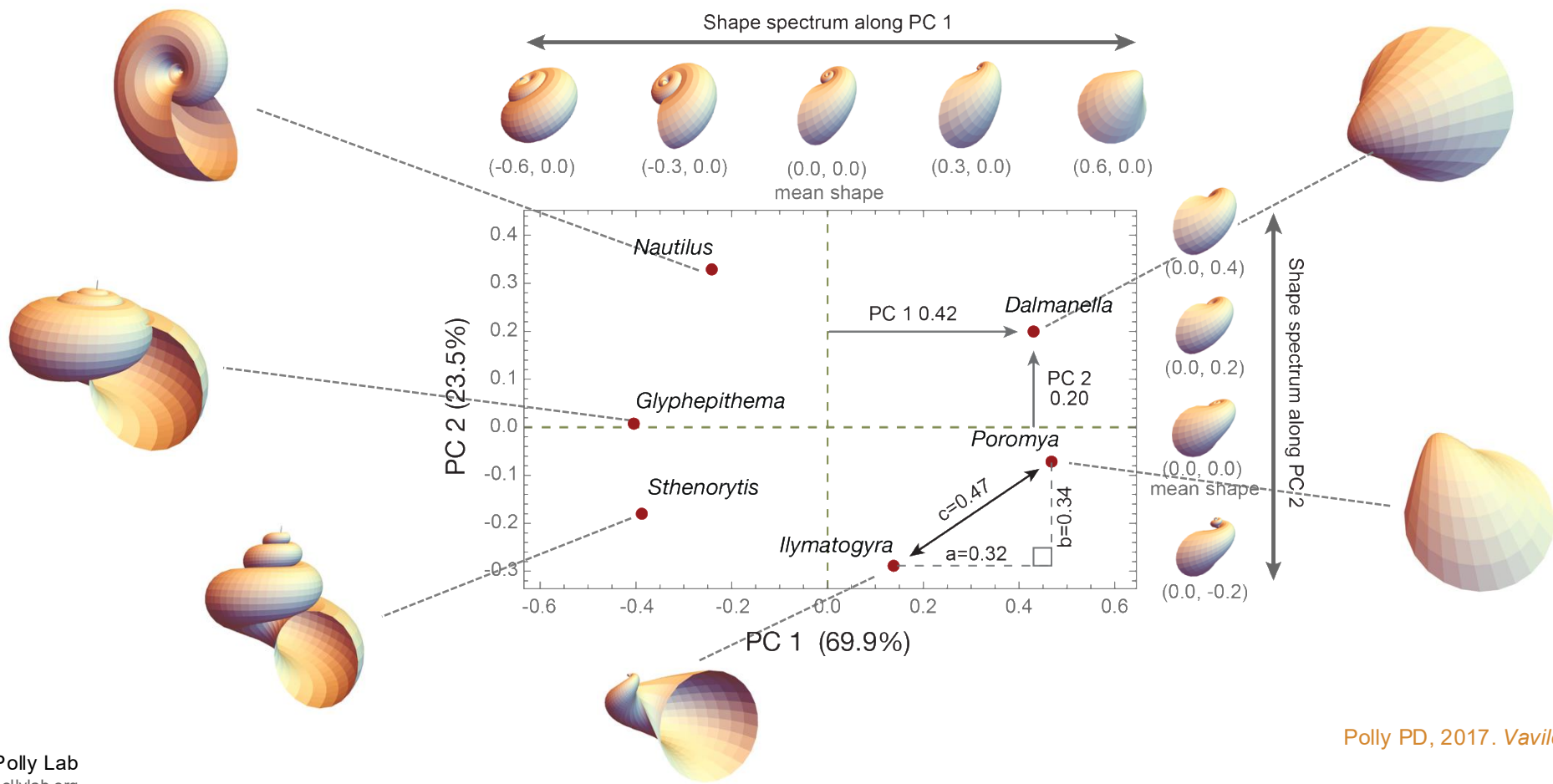


Mean wolf



What are morphospaces?

# A typical geometric morphometric morphospace...



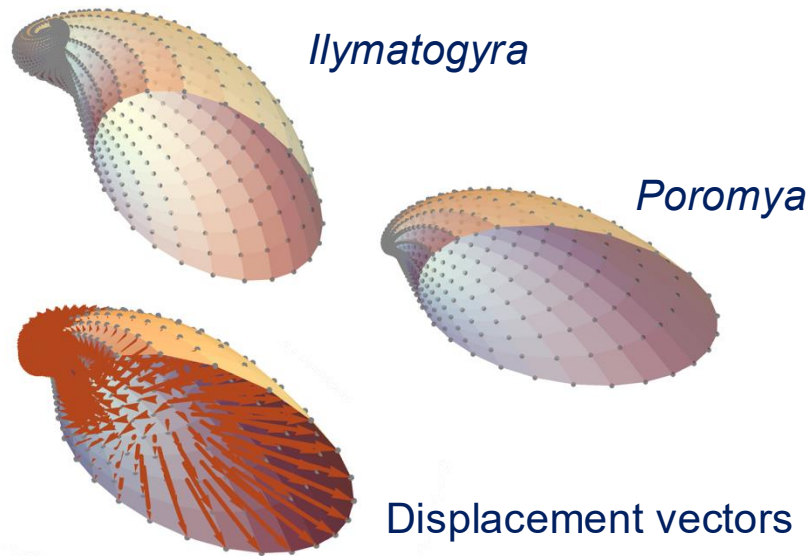
Polly PD, 2017. *Vavilov Journal* 21: 452-461

What are morphospaces?

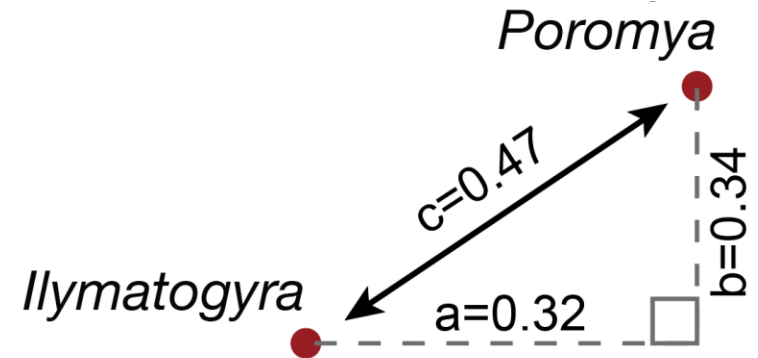
# Distances between objects are preserved

Landmark space

PCA space



=



Procrustes distance = sum of displacements  
between homologous landmarks

Procrustes distance = full multivariate  
distance between objects in morphospace



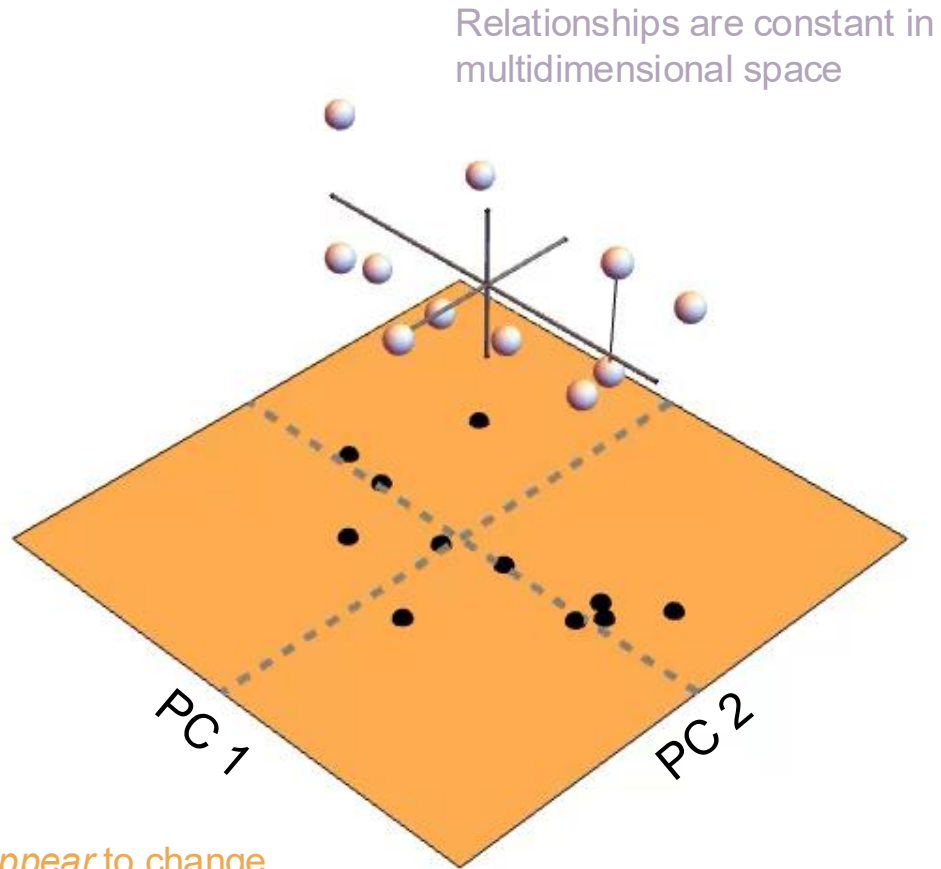


What are morphospaces?

# Distances between objects are preserved

## Geometric morphometric morphospaces are strongly multivariate

- distances between objects in the full space are constant
- PC axes **are** sample dependent
- operations on a small number of PCs **are** sample dependent
- patterns of interest may not be represented on first PC axes
- operations carried out in full multivariate morphospace **are not** sample-dependent



Addition of new specimen simply changes coordinate system, not the relationship between shapes

Relationships *appear* to change on two-dimensional projection

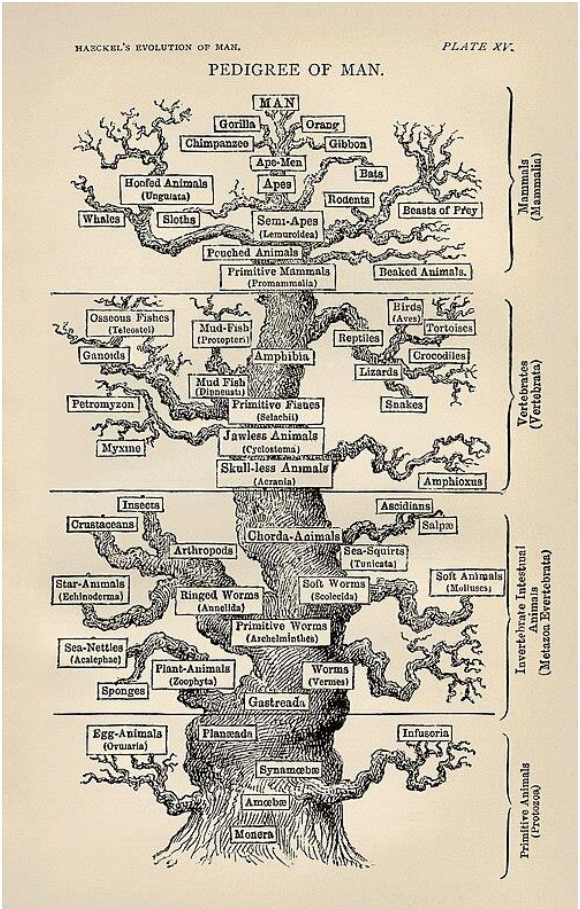


# Phylogenetics

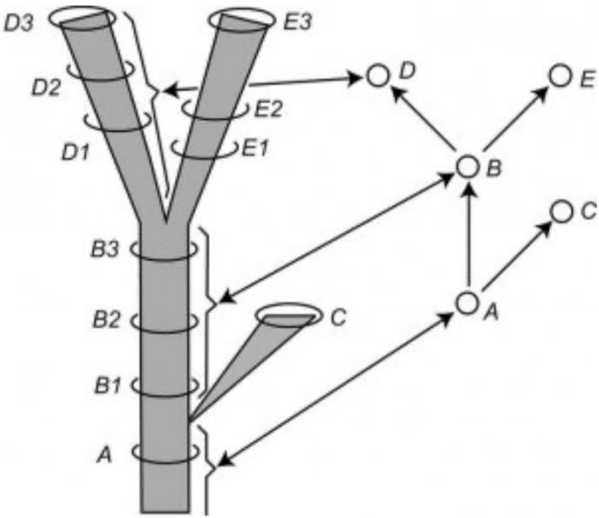
(and its interaction with high dimensional spaces)



Darwin, 1838



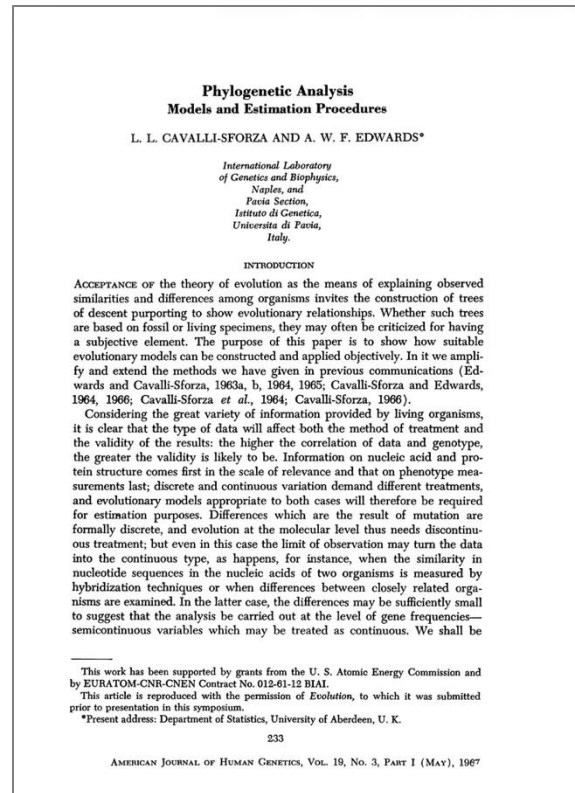
Haeckel, 1879



Hennig, 1966

# Computational phylogeny reconstruction for morphology

Gene frequencies & likelihood  
1965



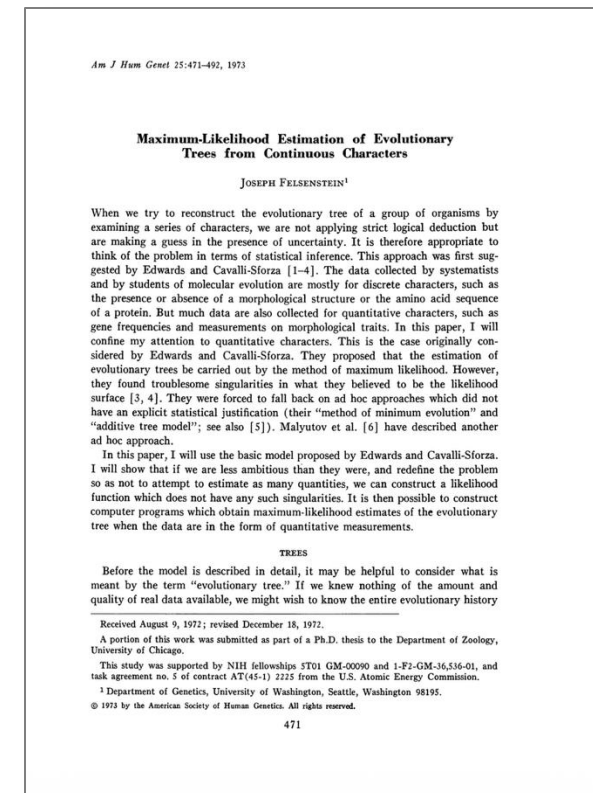
Cavalli-Sforza LL, Edwards AW, 1967.  
Phylogenetic analysis. Models and estimation  
procedures. *Am J Human Genetics*, 19: 233-257.

Discrete traits & parsimony  
1969



Kluge AG, Farris JS. 1969. Quantitative phyletics  
and the evolution of anurans. *Systematic  
Zoology*: 1-32.

Continuous traits & likelihood  
1973

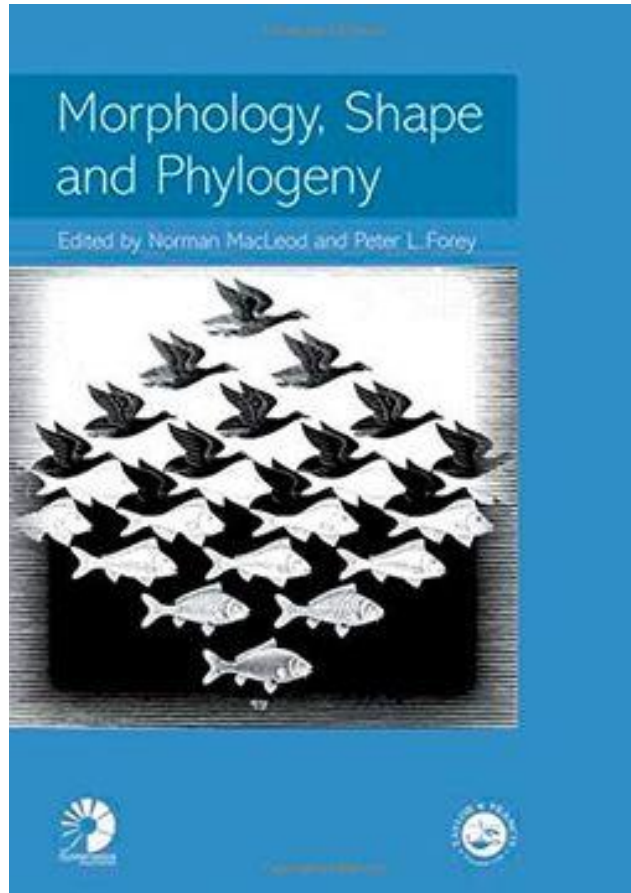


Felsenstein J. 1973. Maximum-likelihood  
estimation of evolutionary trees from continuous  
characters. *Am J Human Genetics*, 25: 471-492.





# Integrating phylogenetics and geometric morphometrics



2002



Systematics Association Annual Meeting  
August 1999  
Glasgow, Scotland





# Phylomorphospace

## The first(?) of geometric morphometrics and statistical phylogenetics

### Chapter 9

### Geometric morphometrics and phylogeny

F. James Rohlf

#### ABSTRACT

This chapter reviews some of the important properties of geometric morphometric shape variables and discusses the advantages and limitations of the use of such data in studies of phylogeny. A method for fitting morphometric data to a phylogeny (i.e., estimating ancestral states of the shape variables) is presented using the squared-change parsimony estimation criterion. These results are then used to illustrate shape change along a phylogeny as a deformation of the shape of any other node on the tree (e.g., the estimated root of the tree). In addition, a method to estimate the digitized image of an ancestor is given that uses averages of unwarped images. An example dataset with 18 wing landmarks for 11 species of mosquitoes is used to illustrate the methods.

Rohlf FJ, 2002, Geometric morphometrics and Phylogeny, pp. 175-193 in *Morphology, Shape, and Phylogenetics*.

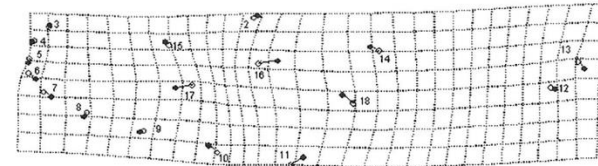


Figure 9.3 Visualization of the estimated shape for ancestor of the Aedini (*Aedes*, *Psorophora*, and *Mansonia*, see Figure 9.2). The grid shows the thin-plate spline transformation from the starting form (●) to the estimated configuration (○). The estimated shape at the root of the tree was used as the starting form.

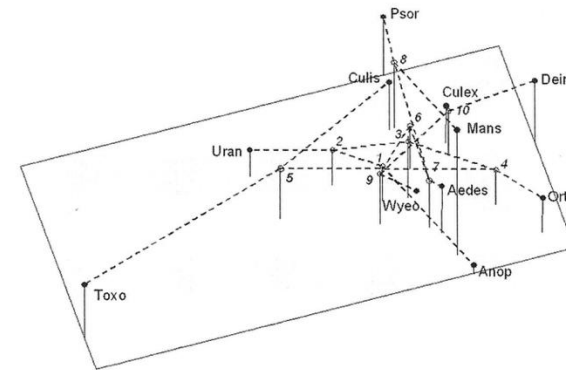


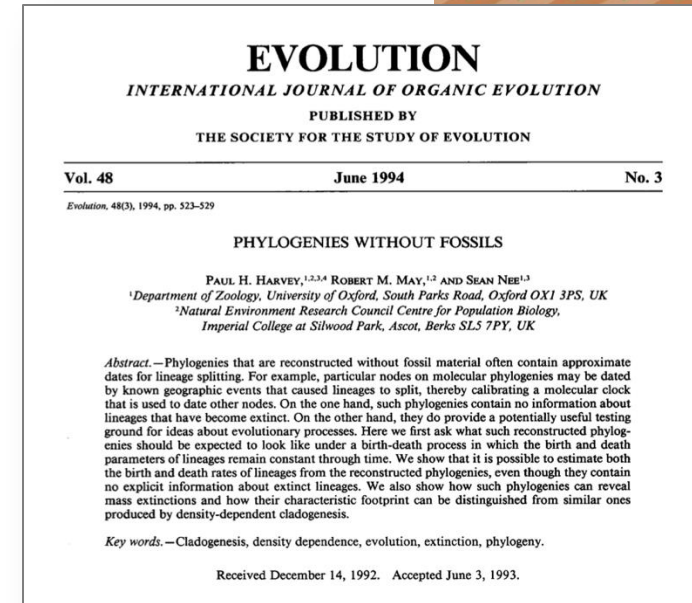
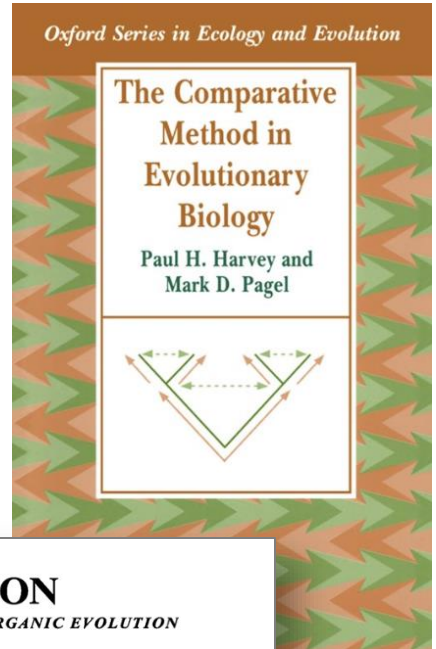
Figure 9.4 Ordination from a non-metric multidimensional scaling analysis of a matrix of distances between all species (●) and estimated internal nodes (○). Phylogenetic tree from Figure 9.2 superimposed using broken lines to link points. OTU codes are given in Table 9.1. Internal nodes numbered in a preorder traversal of the tree beginning with the root. Stress = 0.145. matrix correlation = 0.978.



# Statistical phylogenetics

Probabilistic, likelihood, and Bayesian models for relationships between characters and trees

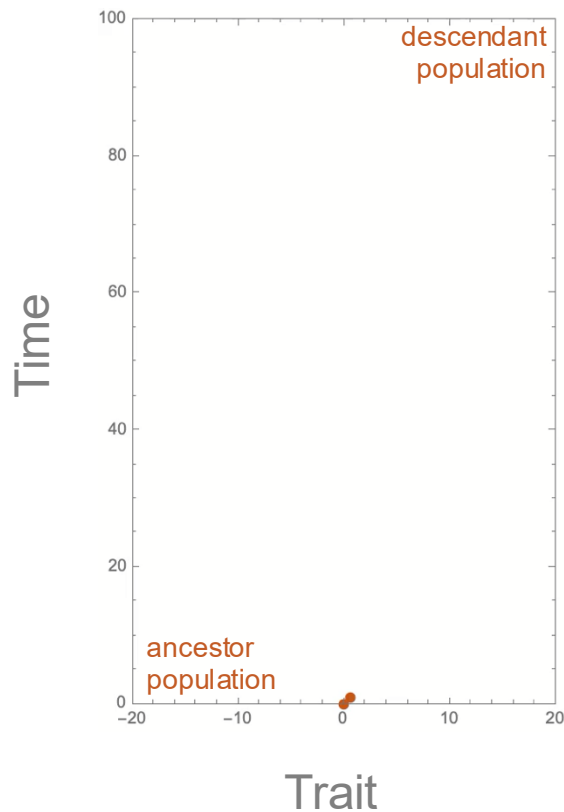
- phylogeny reconstruction
- ancestral state (shape) reconstruction
- estimation of evolutionary rates
- model selection for evolutionary “modes”
- hypothesis testing for rate and “mode” shifts
- testing for evolutionary convergence
- testing for multiple adaptive peaks
- performance trade-off modeling



# Brownian motion is the null model...

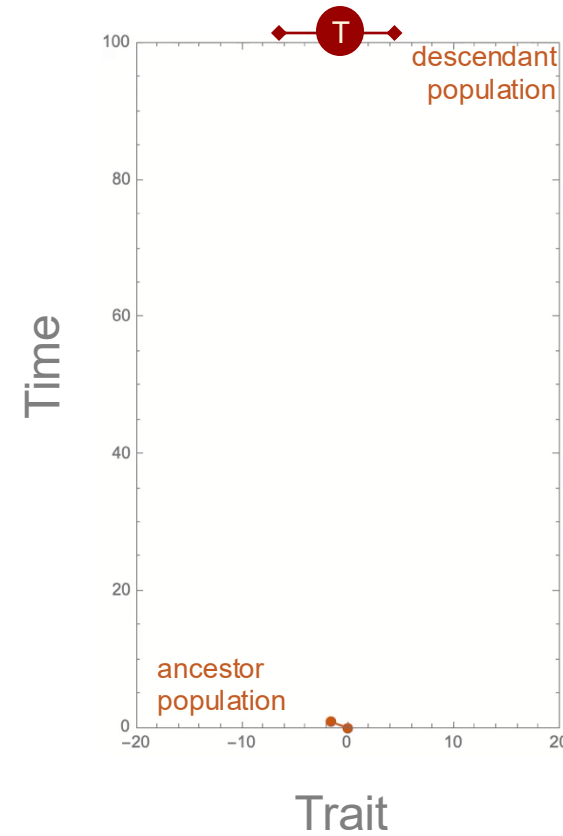
...most other models are juxtaposed to it

Brownian motion  
(genetic or selectional drift)  
in a single lineage



evolution at each step  
is random in direction  
and magnitude

Ornstein-Uhlenbeck  
(stabilizing selection)  
in a single lineage

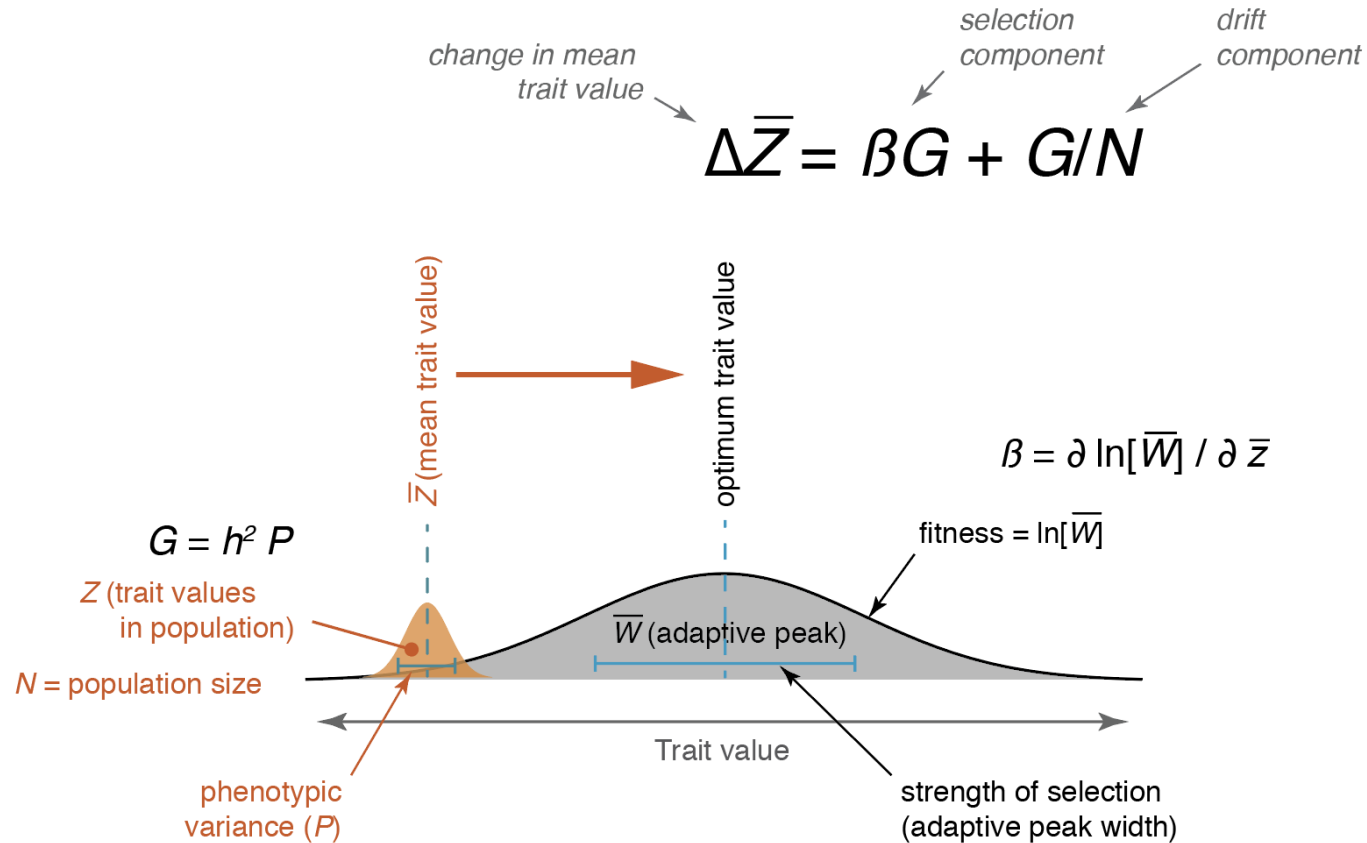


evolution at each step  
depends on distance  
from target  $T$



Nearly all phylogenetic statistical are based on...

## Lande's adaptive peak model for quantitative traits



Lande R, 1976. Natural selection and random genetic drift in phenotypic evolution. *Evolution*, 30: 314-334.



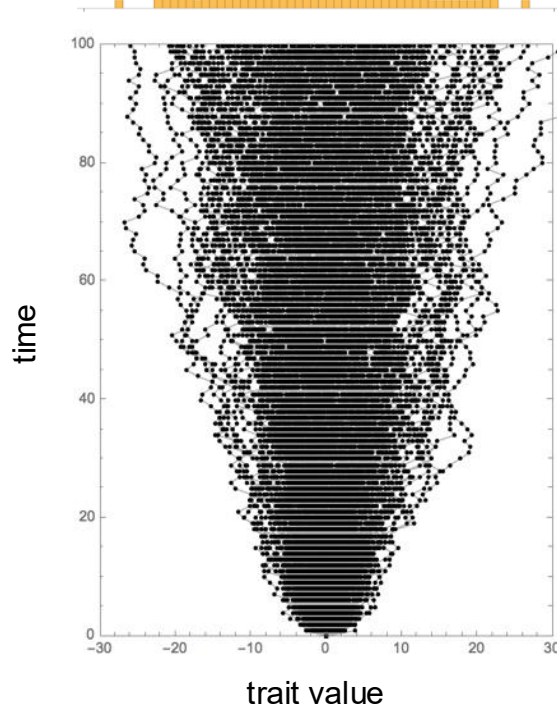


# Likelihood of an evolutionary outcome depends on the model

Best model can be selected based on observed outcomes

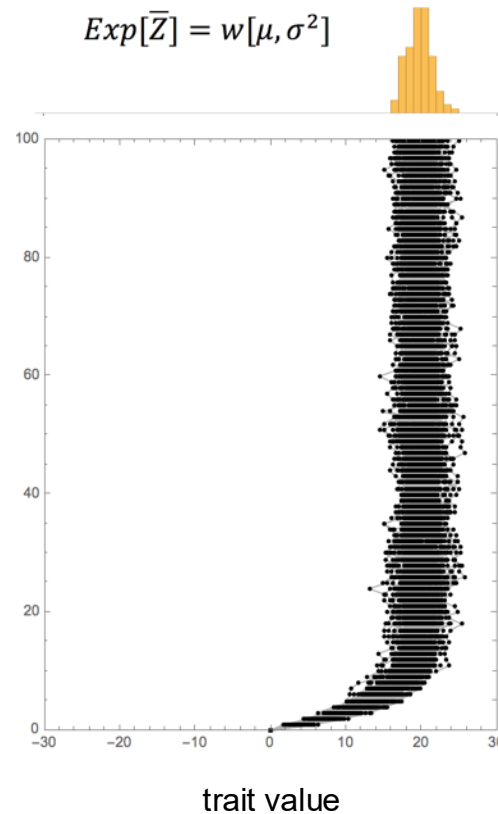
Brownian motion

$$Exp[\bar{Z}] = t\sigma_{\mu=0}^2$$



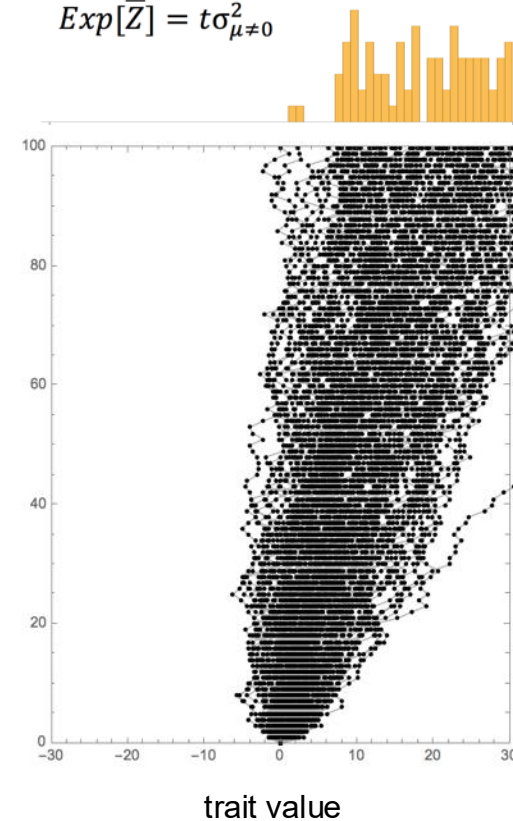
Ornstein-Uhlenbeck (OU)  
or stabilizing selection

$$Exp[\bar{Z}] = w[\mu, \sigma^2]$$



Directional

$$Exp[\bar{Z}] = t\sigma_{\mu \neq 0}^2$$



# Example: model selection in a paleontological lineage

## Gene Hunt

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### Phenotypic Evolution in Fossil Species: Pattern and Process

Gene Hunt<sup>1</sup> and Daniel L. Rabosky<sup>2</sup>

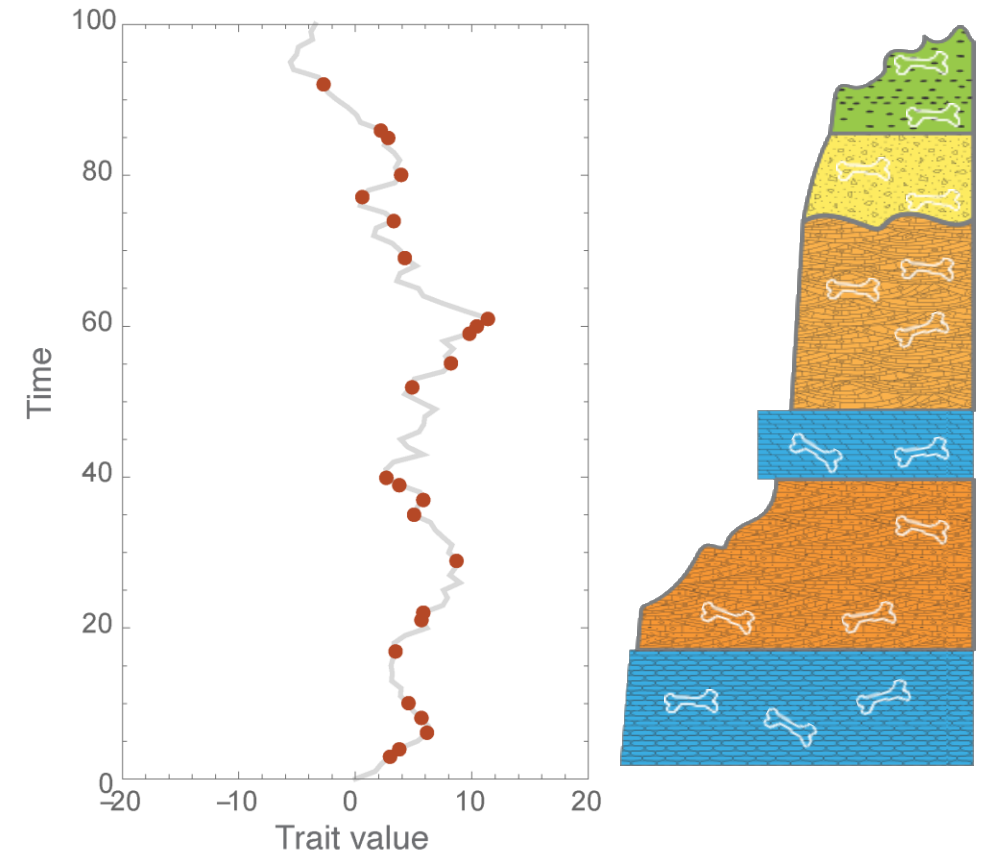
<sup>1</sup>Department of Paleobiology, National Museum of Natural History, Smithsonian Institution, Washington, DC 20560; email: hunt@si.edu  
<sup>2</sup>Department of Ecology and Evolutionary Biology and Museum of Zoology, University of Michigan, Ann Arbor, Michigan 48109; email: drabosky@umich.edu

Annu. Rev. Earth Planet. Sci. 2014. 42:421–41  
The *Annual Review of Earth and Planetary Sciences* is online at earth.annualreviews.org  
This article's doi: 10.1146/annurev-earth-040809-152524  
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**Keywords**  
phenotypic evolution, fossil time series, punctuated equilibrium, speciation, stasis, trends

**Abstract**  
Since Darwin, scientists have looked to the fossil record with the hope of using it to document how the phenotypes of species change over substantial

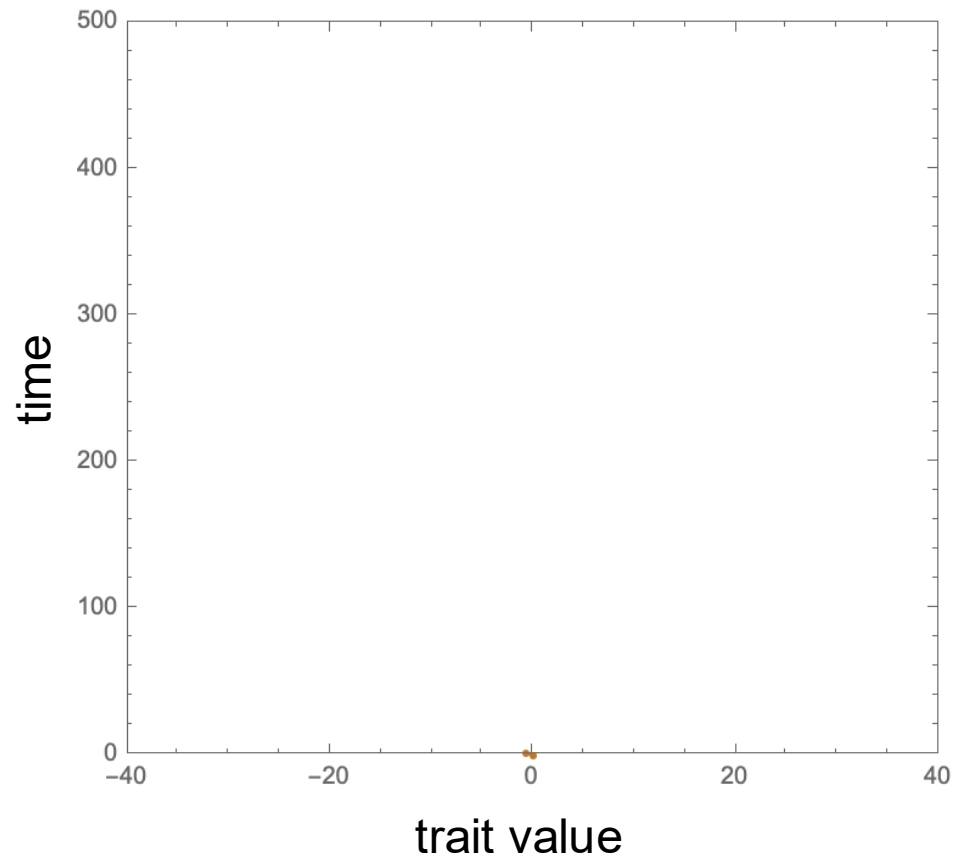
|                                |                        |
|--------------------------------|------------------------|
| Best model                     | URW                    |
| N                              | 39                     |
| $\hat{\mu}_{\text{step}}$      | 0.166                  |
| $\hat{\sigma}^2_{\text{step}}$ | 0.034                  |
| $\hat{\theta}$                 | 1.384                  |
| $\hat{\omega}$                 | 0.004                  |
| AIC <sub>c</sub> URW           | -124.270               |
| AIC <sub>c</sub> GRW           | -122.860               |
| AIC <sub>c</sub> Stasis        | -96.080                |
| wt URW                         | 0.670                  |
| wt GRW                         | 0.330                  |
| wt Stasis                      | $5.064 \times 10^{-7}$ |



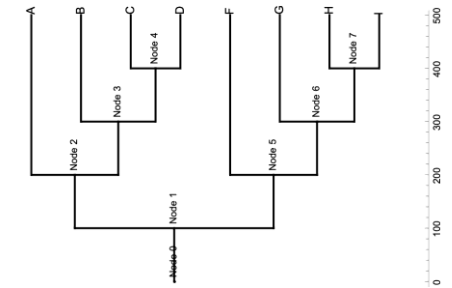
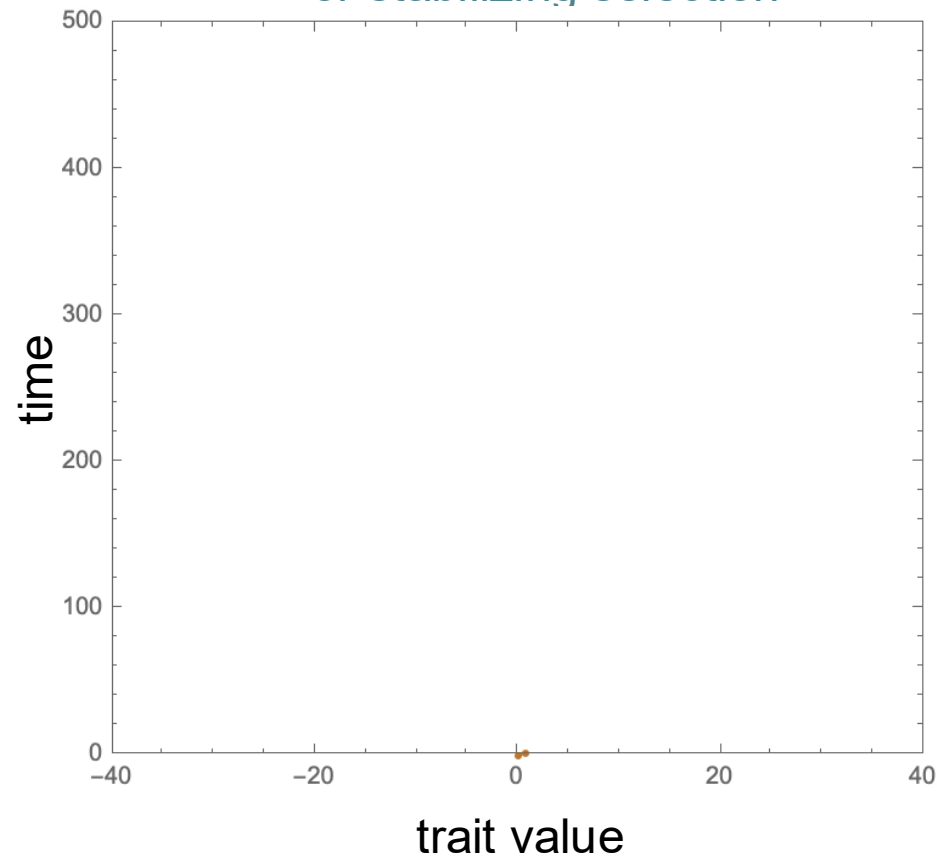
# Statistical models of evolution on phylogenetic trees

Same idea, but tip values covary by shared ancestry

Brownian motion



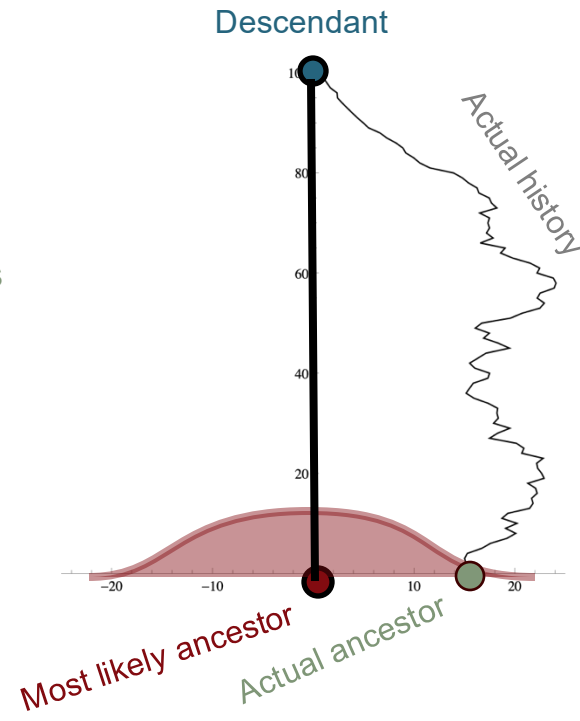
Ornstein-Uhlenbeck (OU)  
or stabilizing selection



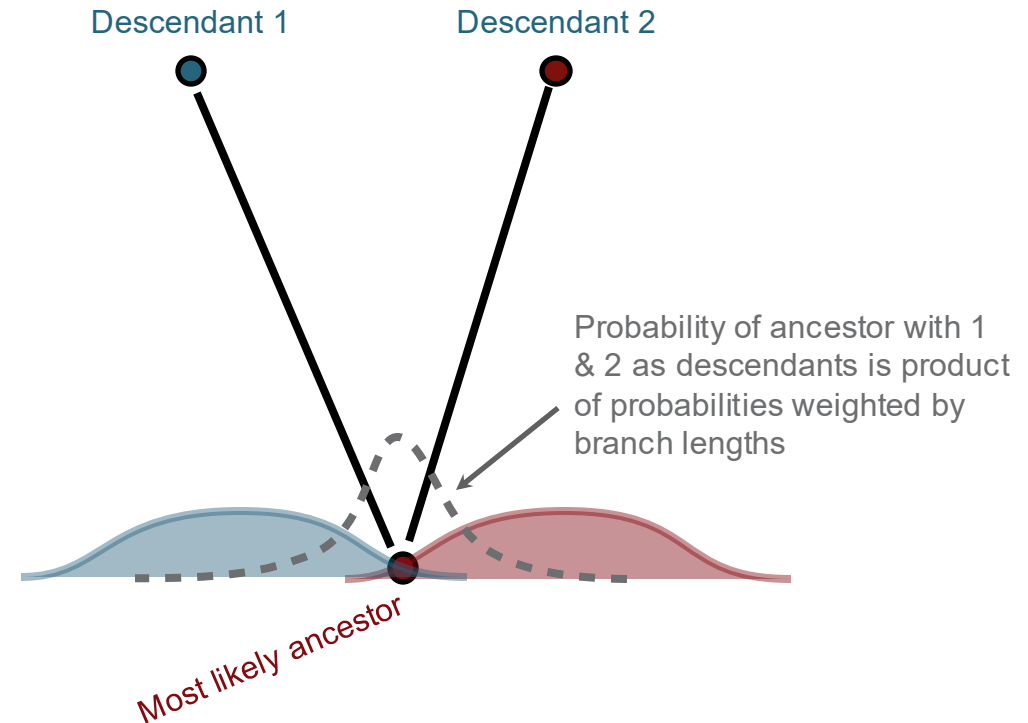
# Ancestor reconstruction using a BM model

## For one descendant

- For one tip, the most likely ancestor value is the same
- Variance in likelihood is proportional to time elapse
- Most likely path is a straight line

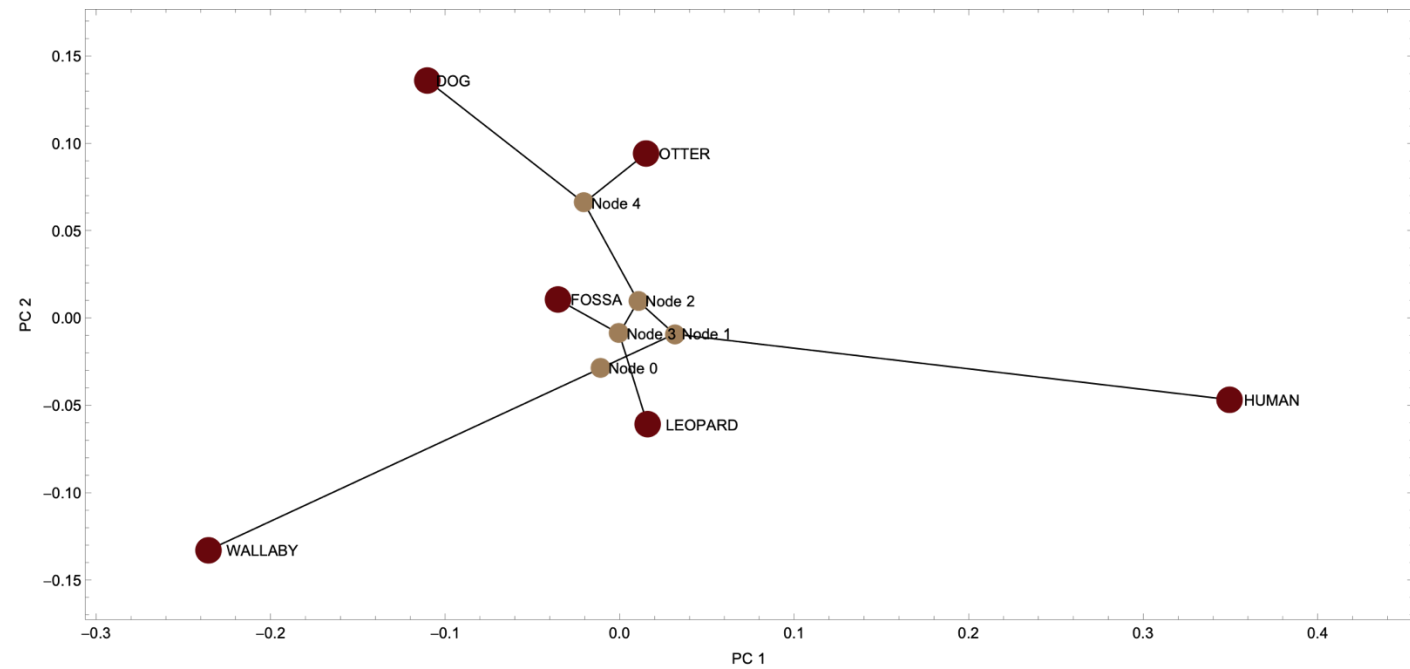
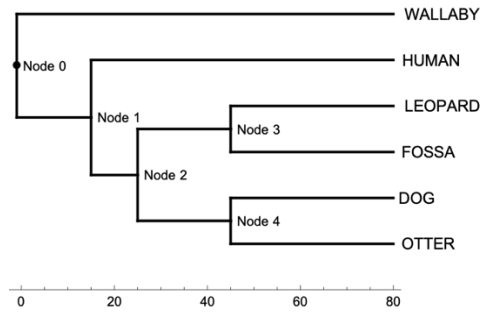


## For two or more descendants



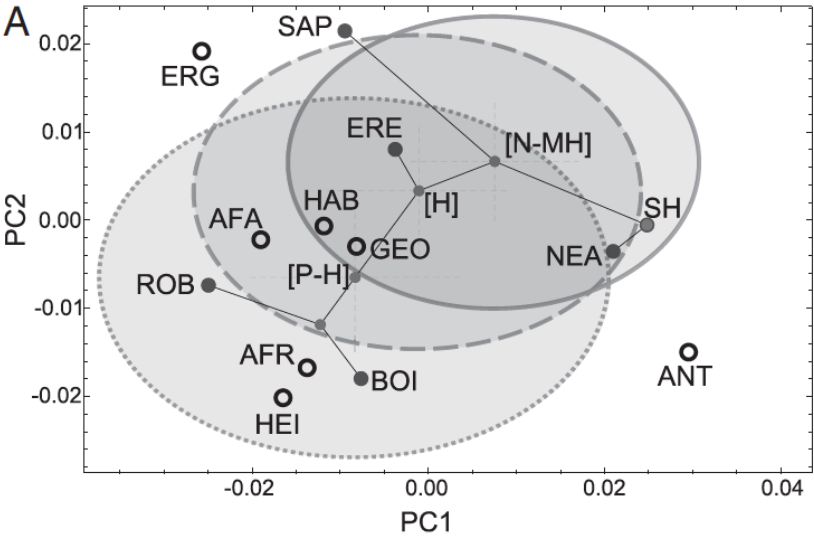
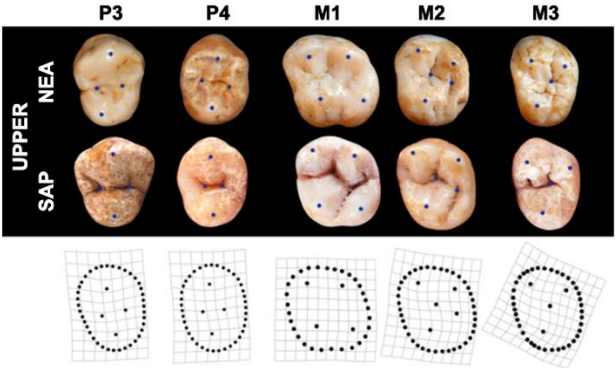
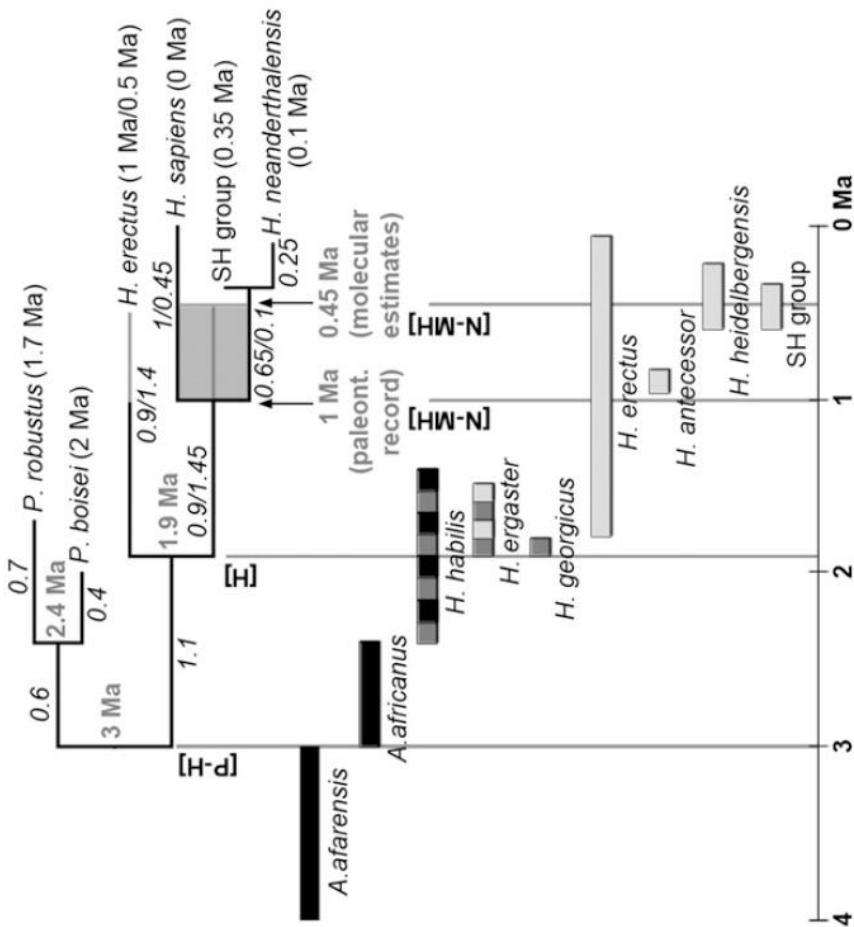


# Phylomorphospaces are based on these models



# No known hominin species matches the expected dental morphology of the last common ancestor of Neanderthals and modern humans

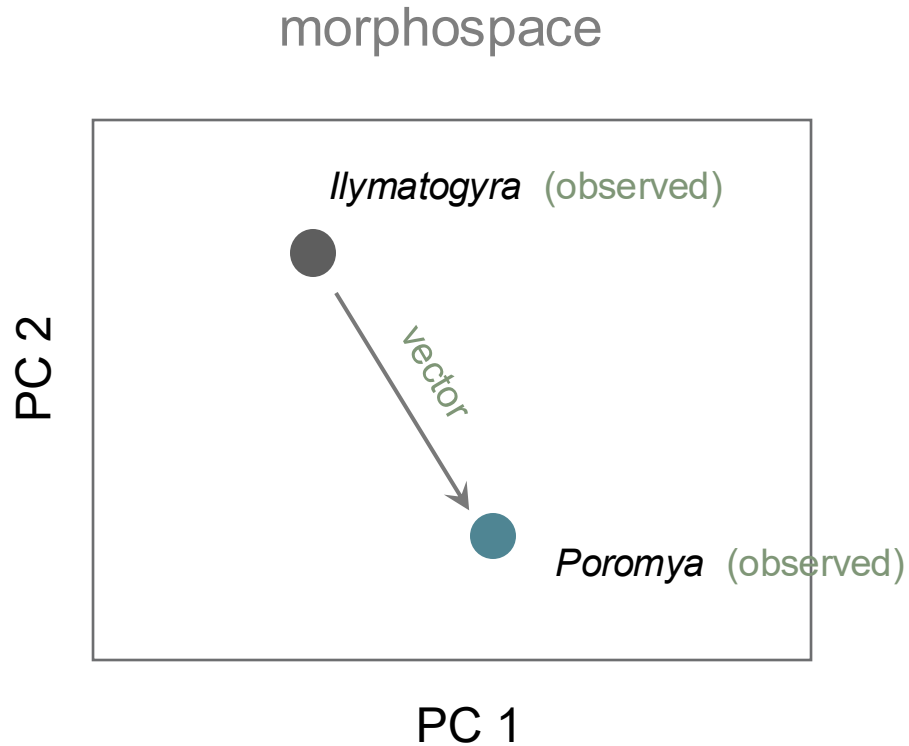
Aida Gómez-Robles<sup>a,b,1</sup>, José María Bermúdez de Castro<sup>c</sup>, Juan-Luis Arsuaga<sup>d</sup>, Eudald Carbonell<sup>e</sup>, and P. David Polly<sup>f</sup>



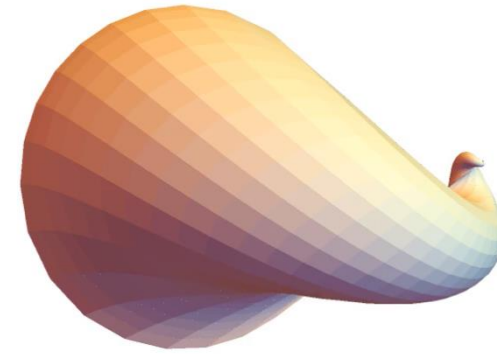
Example: `ProbabilitiesOfShapesAsAncestors[proc, labels, tree]`

|                   | Node 0        | Node 1        | Node 2        | Node 3        | Node 4        | Node 5 |
|-------------------|---------------|---------------|---------------|---------------|---------------|--------|
| A.africanus       | <b>0.8920</b> | <b>0.7500</b> | 0.0249        | 0.0130        | 0.0002        | 0.0000 |
| H.habilis         | <b>0.6460</b> | <b>0.9110</b> | <b>0.2520</b> | <b>0.6490</b> | <b>0.2460</b> | 0.0009 |
| H.ergaster        | 0.0392        | 0.0022        | 0.0000        | 0.0002        | 0.0001        | 0.0000 |
| H.georgicus       | <b>0.3120</b> | 0.0099        | 0.0000        | 0.0001        | 0.0000        | 0.0000 |
| H.antecessor      | 0.0109        | 0.0008        | 0.0000        | 0.0001        | 0.0000        | 0.0000 |
| H.heidelbergensis | 0.0001        | 0.0000        | 0.0000        | 0.0000        | 0.0000        | 0.0000 |

# Do straight lines represent realistic biological paths?



morphologies along path



Empty regions of morphospace....



selection → species



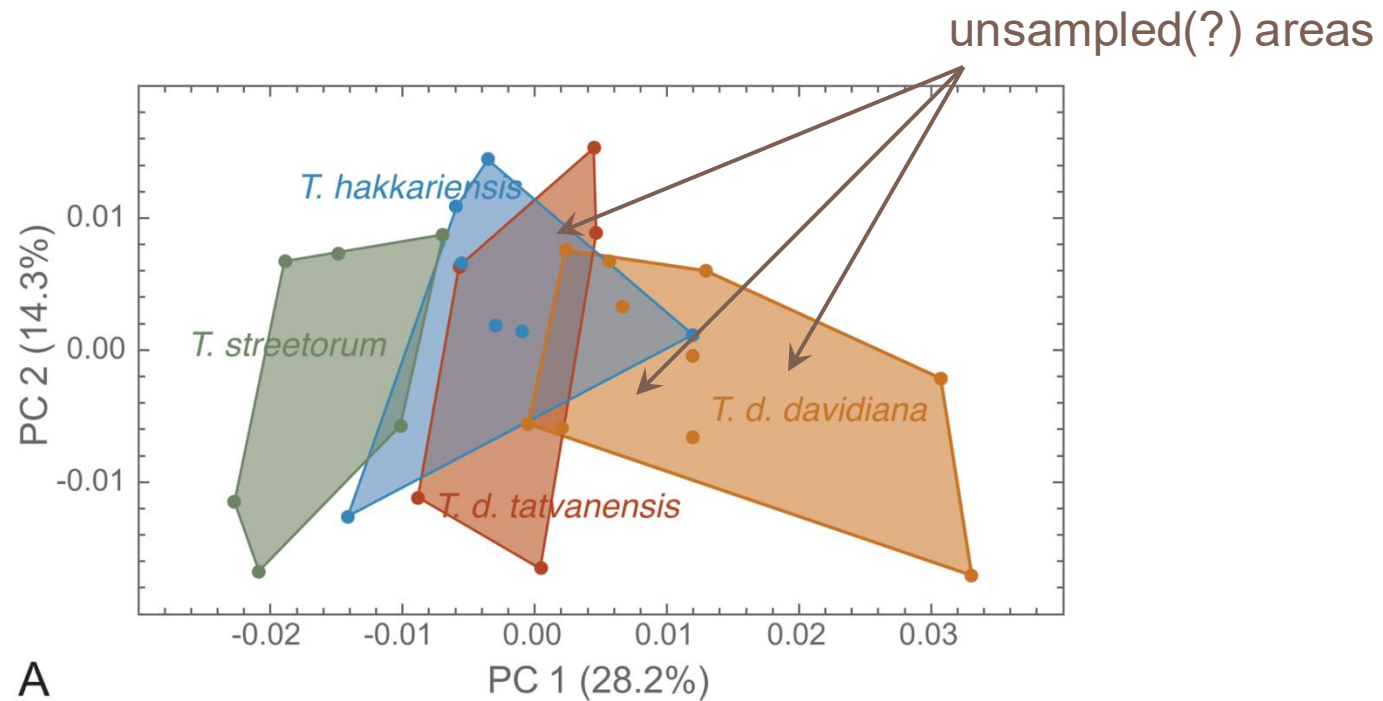
... do not always follow straight paths

# Empty areas of morphospace – Type 1

## Ordinary gaps



*Talpa hakkariensis*



Gündüz et al., 2023, *Zool J Linn Soc*, 199: 567-593



# Empty areas of morphospace – Type 2

“Kendall curvature”

Mathematical topology constraints

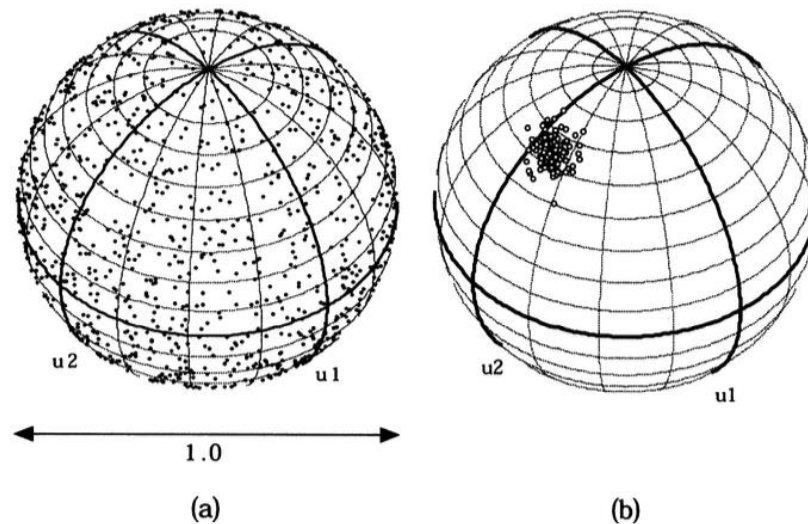


FIGURE 1. A view of Kendall's shape space for triangles showing (a) the mapping of 2,000 random triangles generated by the independent, normal displacement of triplets of points from the origin and (b) the mapping of 110 triangles describing the overall shape of gorilla scapulae. The north pole in both plots corresponds to an equilateral triangle. The south pole corresponds to the reflection of the triangle at the north pole. The equator of each sphere corresponds to triangles with collinear vertices. Longitude is defined with respect to Bookstein's (1996a) linearized Procrustes estimates of uniform shape differences,  $u_1$  and  $u_2$ .

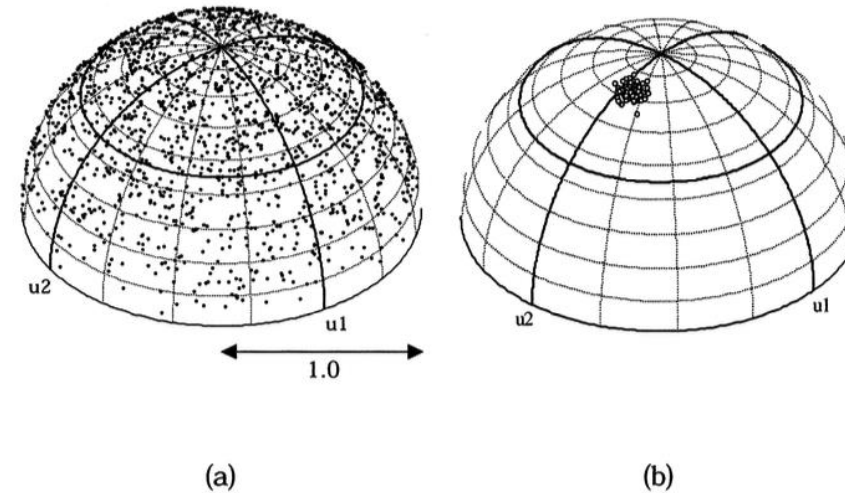
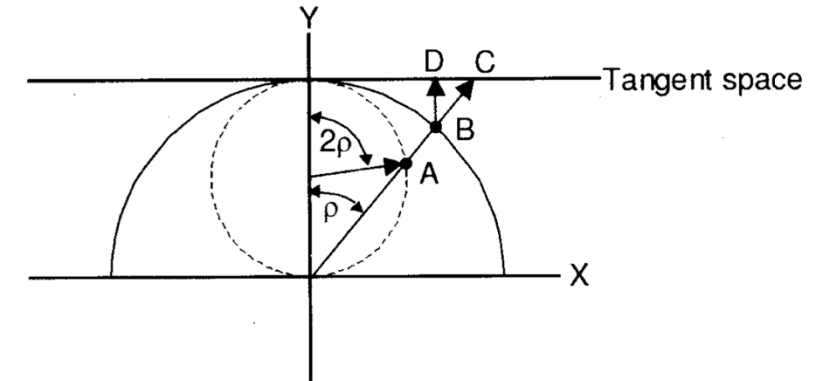


FIGURE 2. A view of the hemisphere of Procrustes-superimposed triangles showing (a) the mapping of the 2,000 random triangles from Figure 1a and (b) the mapping of the 110 gorilla scapulae from Figure 1b. Both sets of data were fit by using an equilateral triangle (north pole) as the reference configuration. The heavy latitudinal line on each hemisphere corresponds to triangles with collinear vertices that map to the equators of the spheres in Figure 1. The equator of the hemisphere corresponds to the reflection of the equilateral triangle at the north pole. Longitude is defined with respect to Bookstein's (1996a) linearized Procrustes estimates of uniform shape differences,  $u_1$  and  $u_2$ .

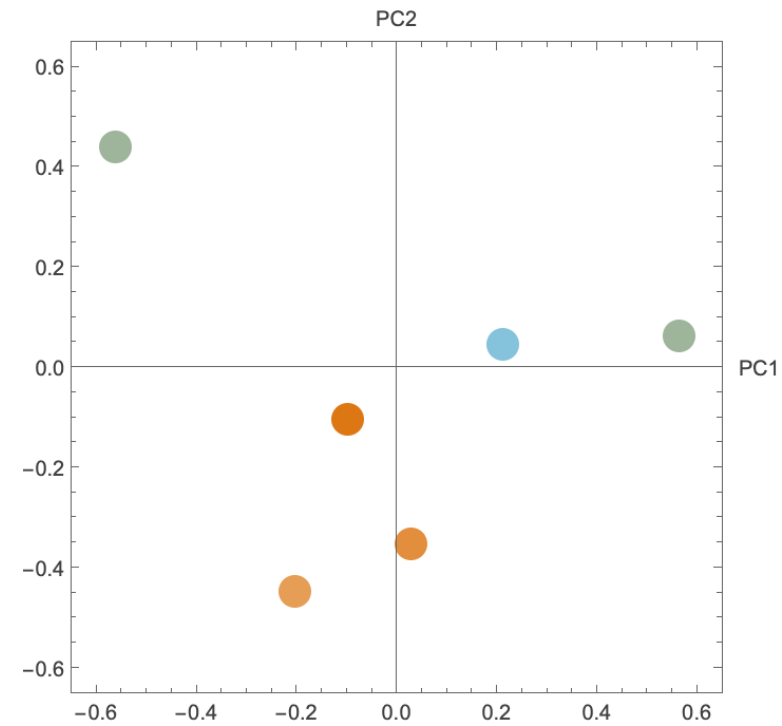
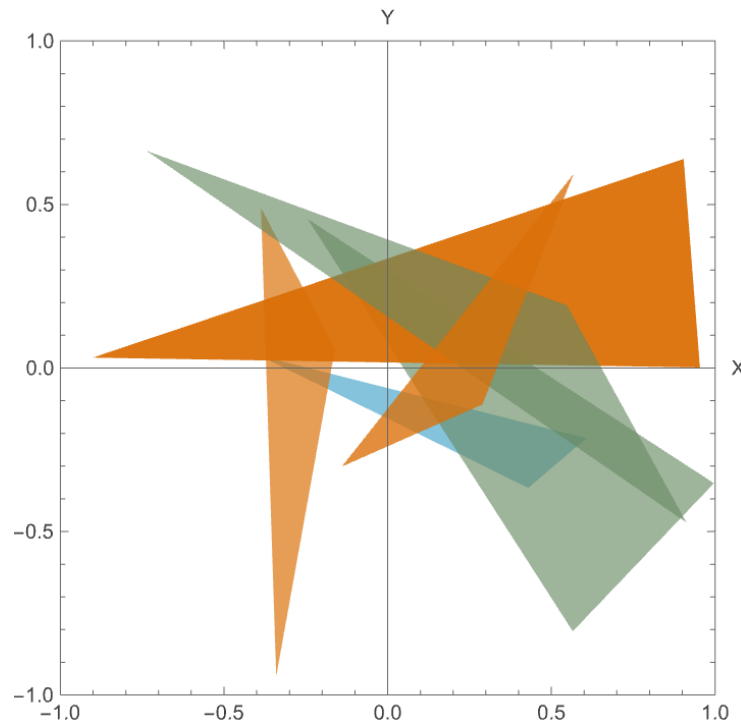
Slice, 2001, "Landmark coordinates aligned by Procrustes analysis do not lie in Kendall's shape space", *Syst Biol* 50: 141-149



# Empty areas of morphospace – Type 2

“Kendall curvature”

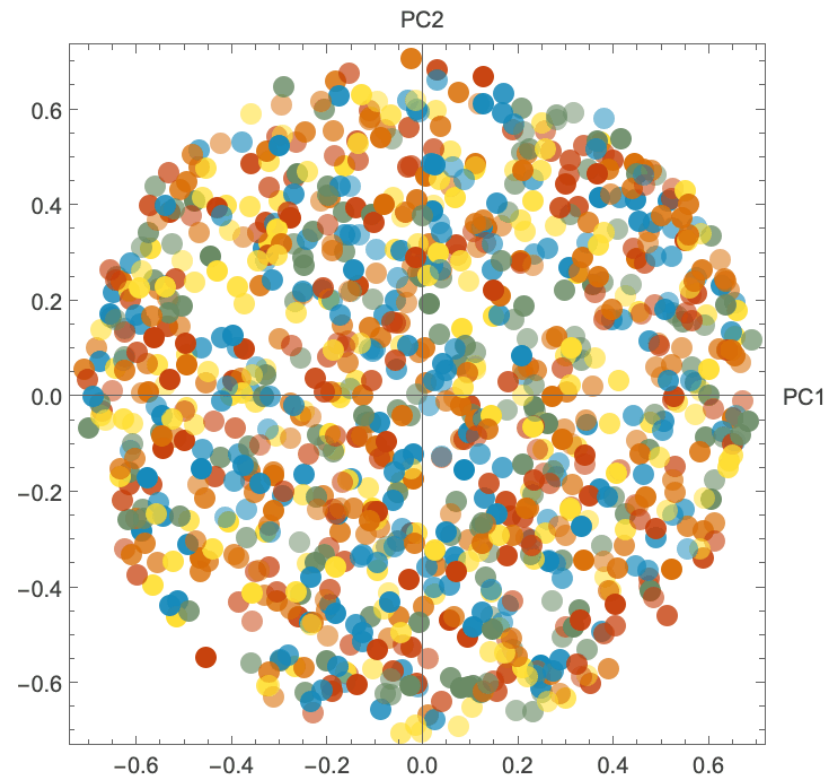
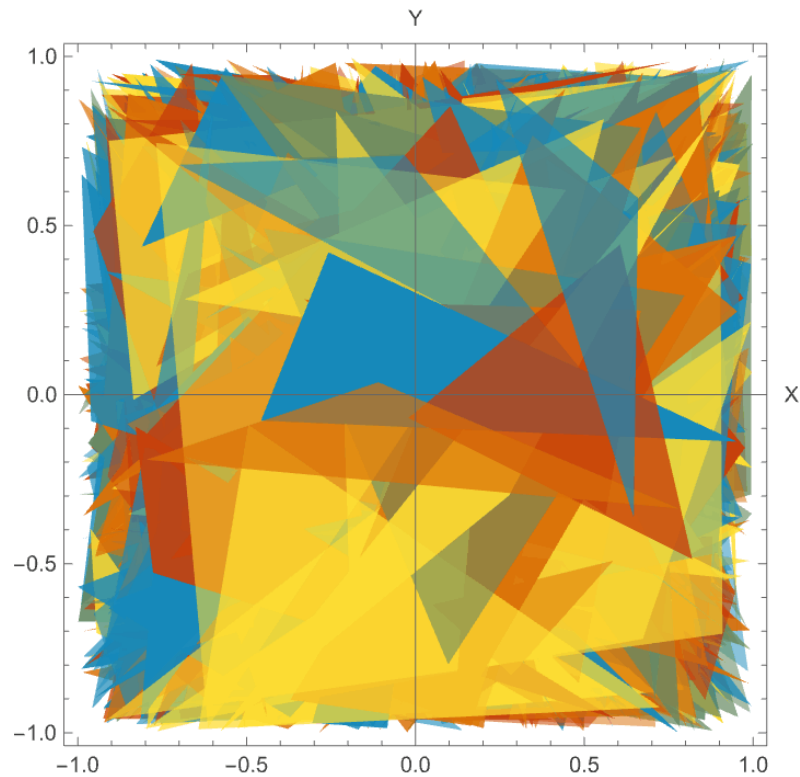
Mathematical topology constraints



# Empty areas of morphospace – Type 2

“Kendall curvature”

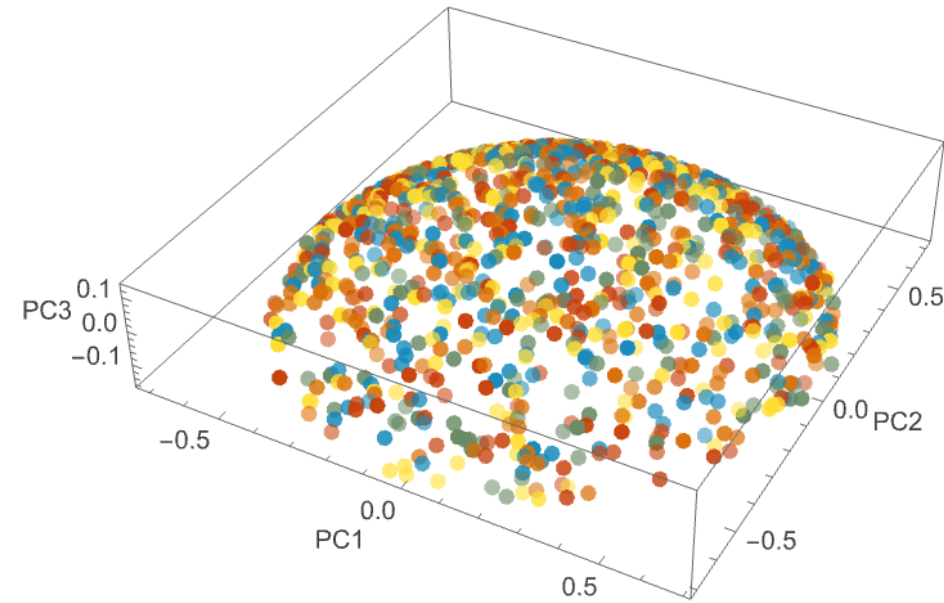
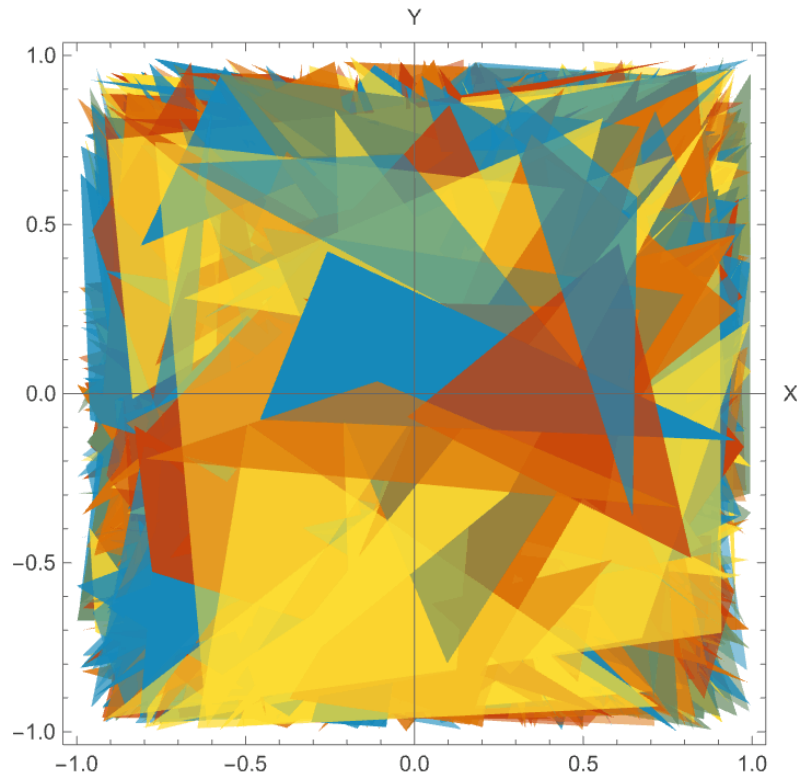
Mathematical topology constraints



# Empty areas of morphospace – Type 2

“Kendall curvature”

Mathematical topology constraints



# Empty areas of morphospace – Type 3

## “Bookstein bends”

Mathematical interactions between shape gradients and “major axes”

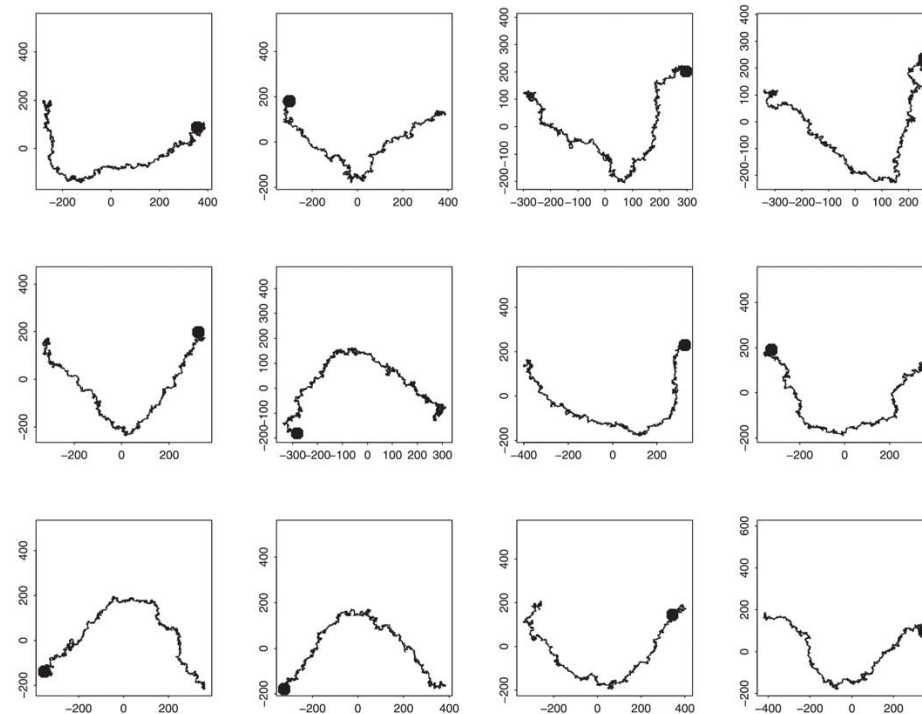


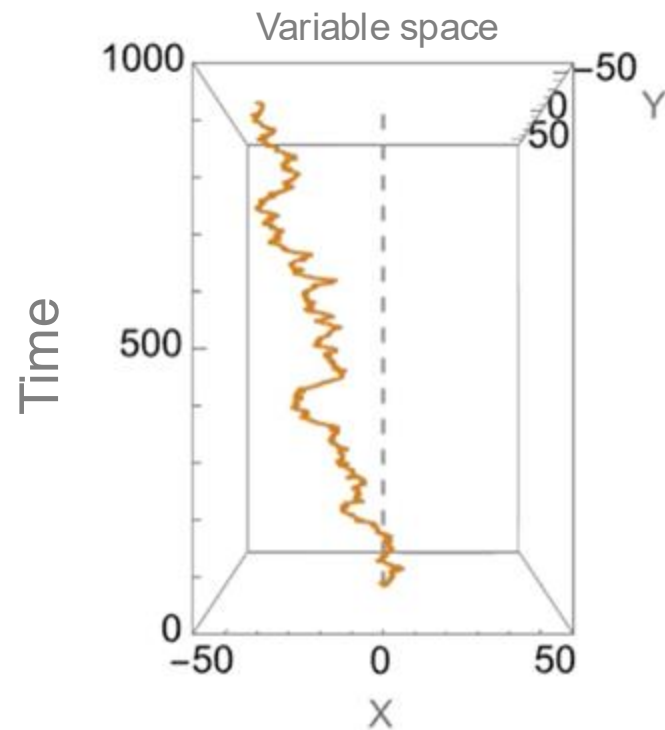
FIGURE 6. First two principal components of twelve isotropic 50-dimensional random walks (Kendall's model for diffusion in shape space). These each had a 50-dimensional sphere of directions for  $PC_1$  to choose from, and given  $PC_1$  a 49-dimensional sphere of directions for  $PC_2$ ; yet the resulting principal component analyses all look more or less the same. All walks are of length 10,000. Axes are as in Figure 5.



# Empty areas of morphospace – Type 3

“Bookstein bends”

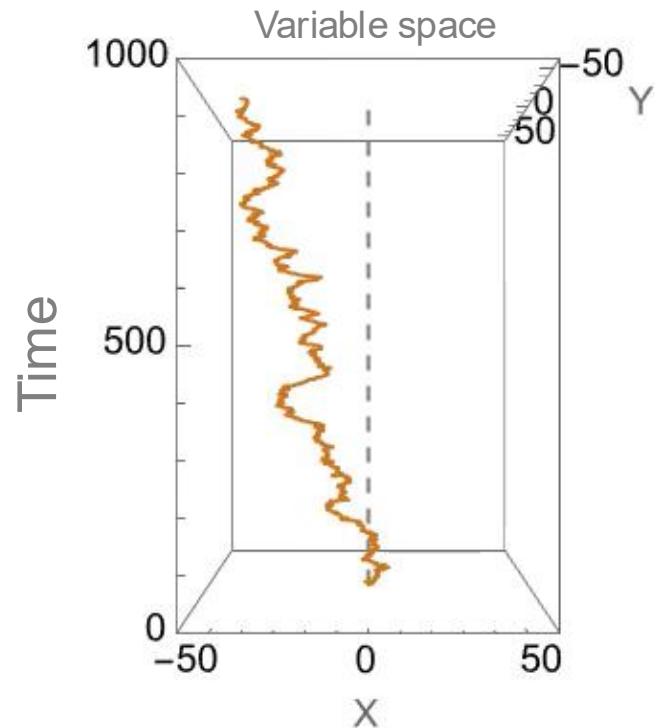
Evolutionary Random Walk  
(2 of 56 variables)



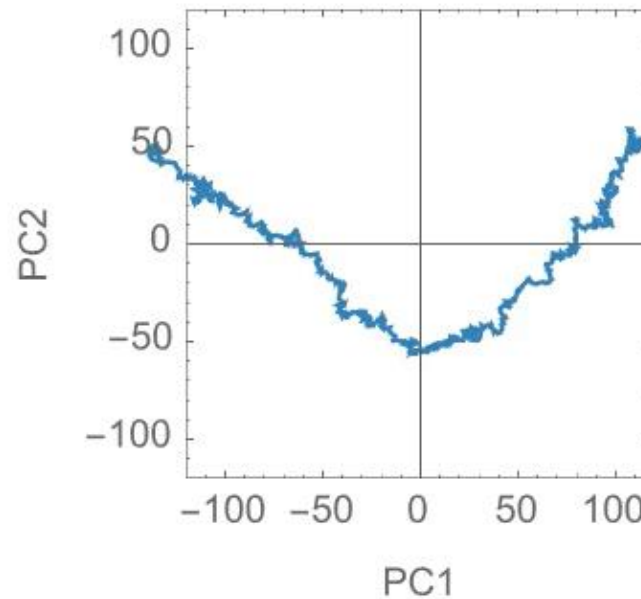
# Empty areas of morphospace – Type 3

“Bookstein bends”

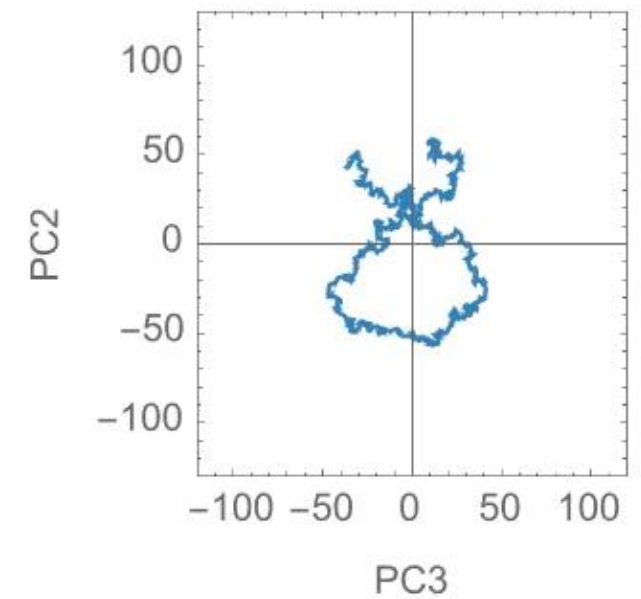
Evolutionary Random Walk  
(2 of 56 variables)



PC morphospace  
(1<sup>st</sup> two dimensions)

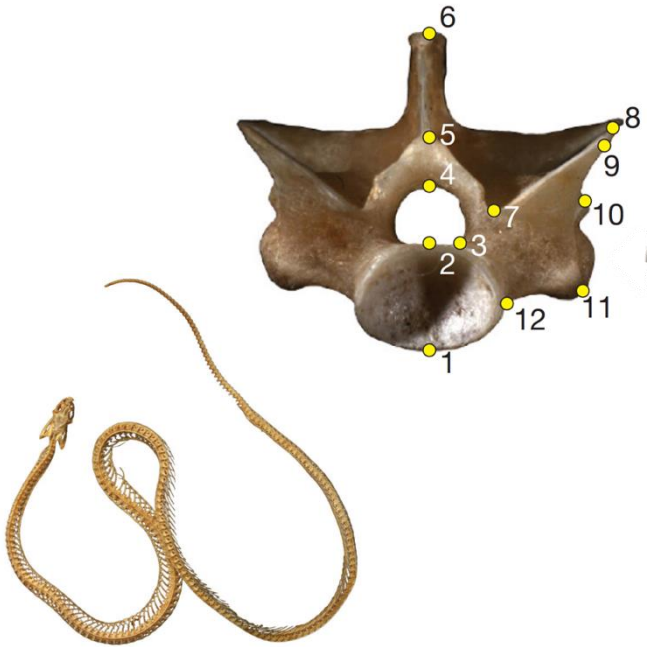


PC morphospace  
(2<sup>nd</sup> two dimensions)

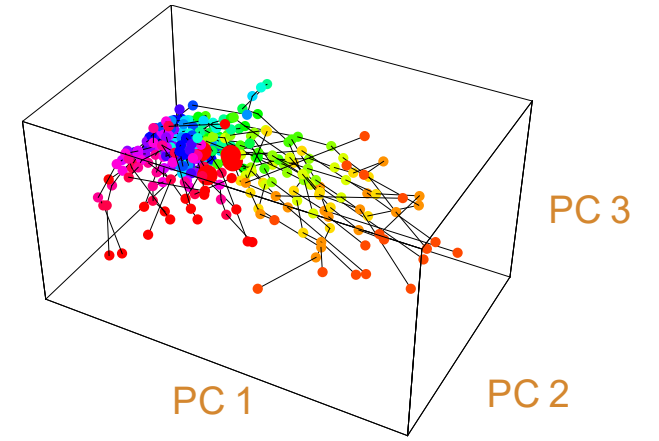
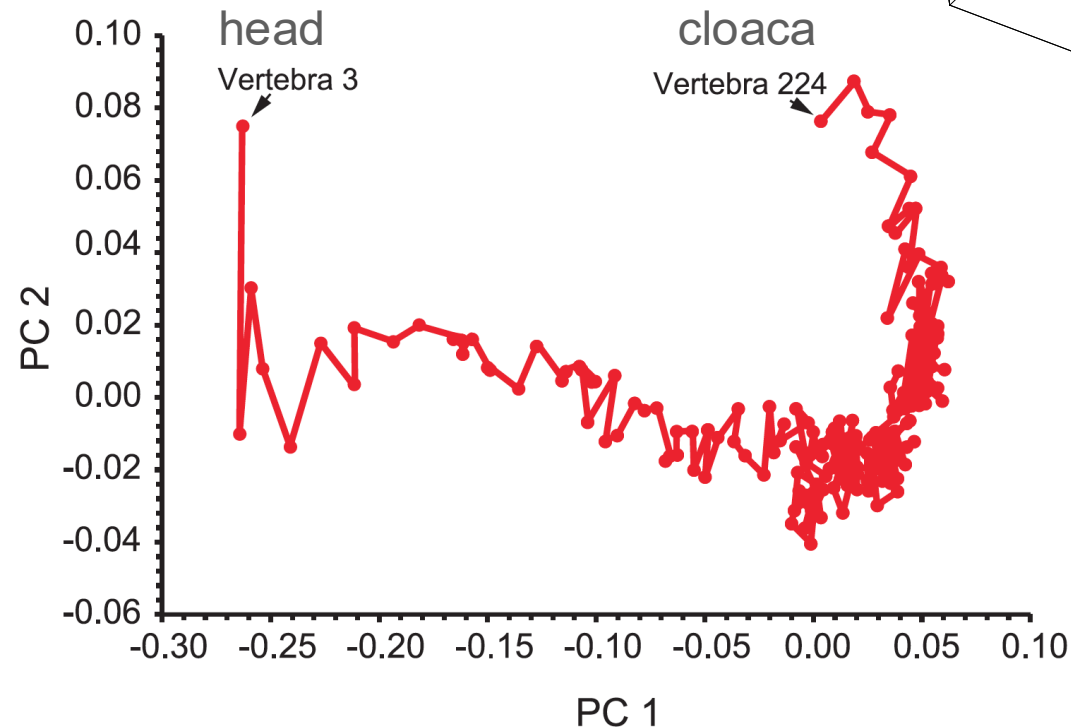


# Example Type 3 : morphological series

Jason J. Head – snake vertebral regionalization

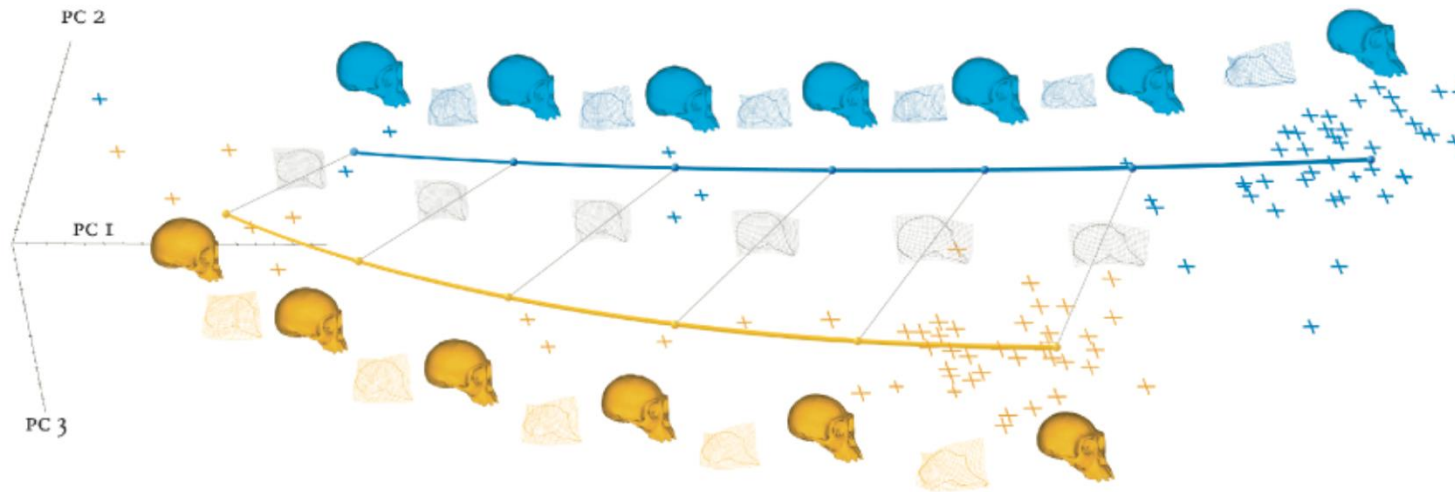


Head & Polly, 2015, *Nature* **520**: 86-89



# Example Type 3: ontogenetic series

Philipp Mitteröcker –growth and allometry differences in *chimps*



Mitteroecker et al., 2005. *Evolution & Development*, 7: 244-258.

# Example Type 3: paleontologic study of group origins

Timothy B. Rowe – phylogenetic origin of Mammalia

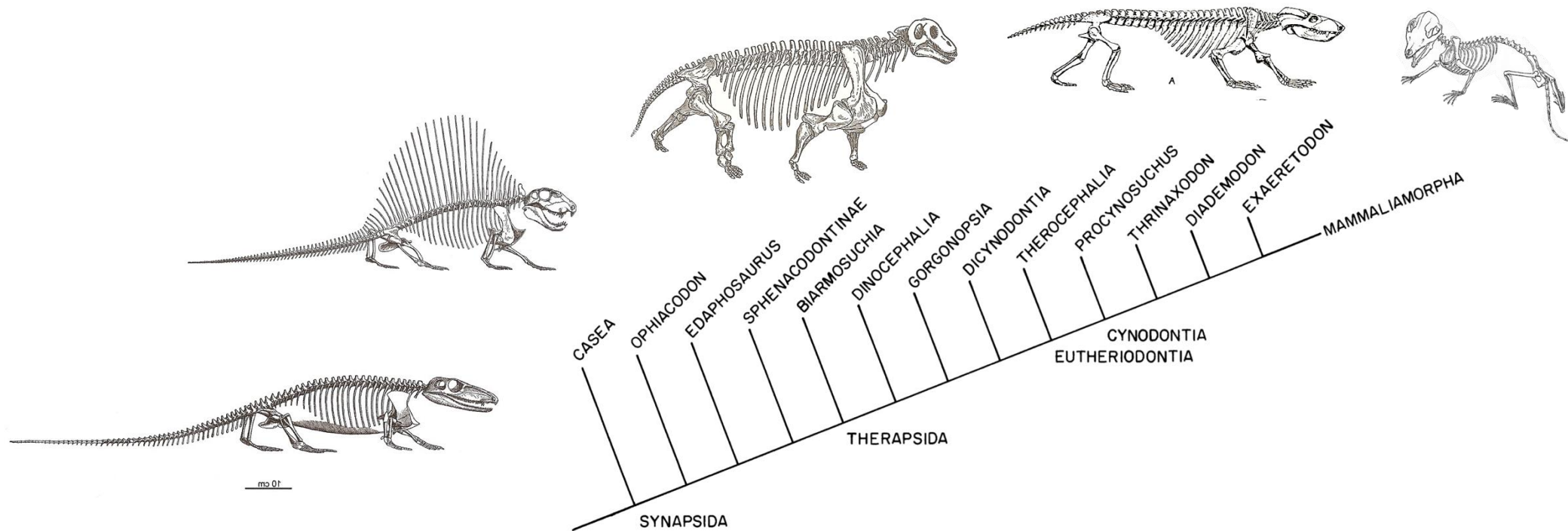
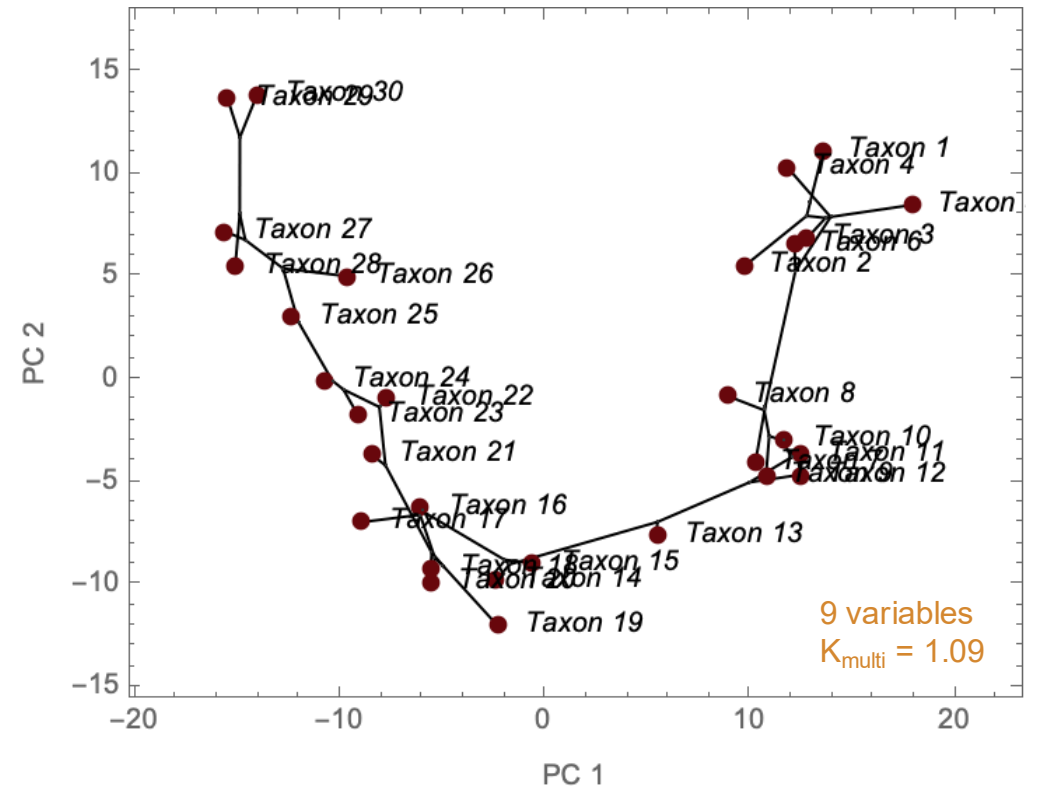
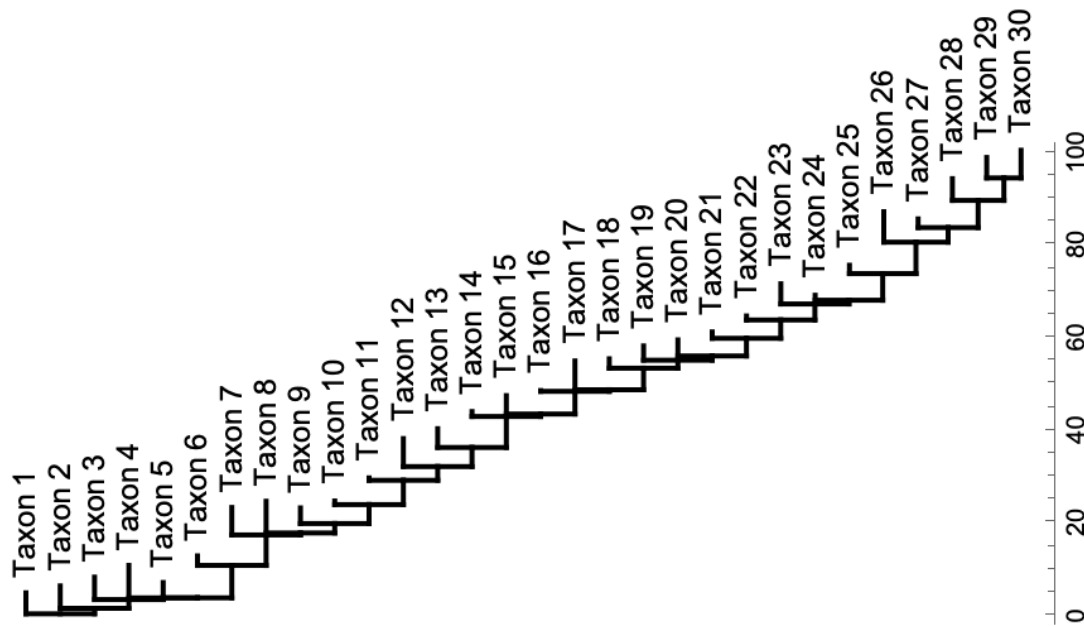


FIGURE 2. Phylogeny of higher systematic categories of Synapsida. This hypothesis depicts the consecutive outgroups used to determine ancestral states for the terminal taxa in this analysis. Character data for this hypothesis are discussed in Rowe (1986a) and Gauthier et al. (1988a).





# Pectinate trees likely to create “Bookstein bends”



# Empty areas of morphospace – Type 4

“Constraint chasms”

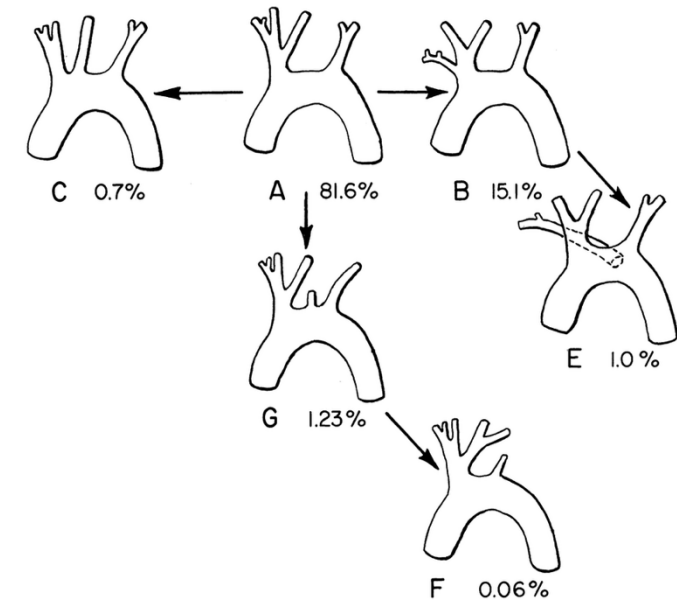
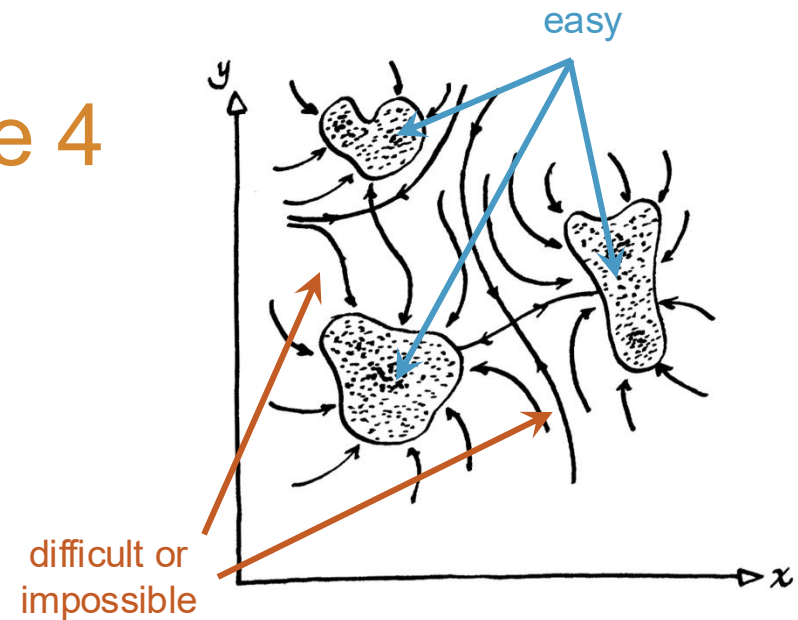
AMER. Zool., 20:653–667 (1980)

## Ontogenesis and Morphological Diversification<sup>1</sup>

PERE ALBERCH<sup>2</sup>

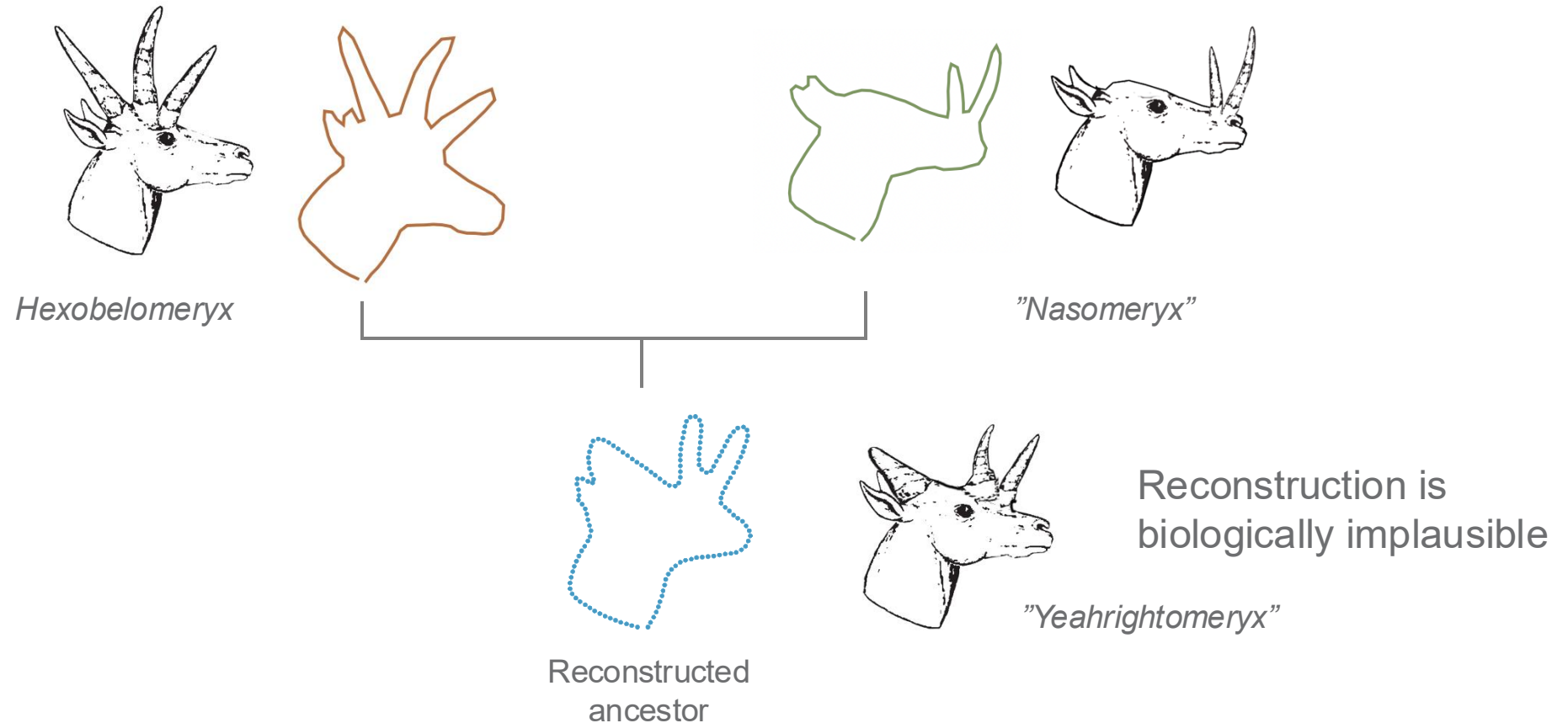
*Museum of Vertebrate Zoology and Department of Zoology,  
University of California, Berkeley, California 94720*

**SYNOPSIS.** The role of development in constraining the directionality and patterns of morphological evolution is examined. The nature of morphological variation and appearance of morphological novelties is determined by the epigenetic properties of the organism. Consideration of these properties has profound implications for current theories of morphological evolution. Developmental constraints impose severe limitations on the gradualistic action of directional selection. Evolution is viewed as the result of differential survival of morphological novelties. However, the production of morphological novelties by developmental programs is not random. This non-randomness in morphologically expressed genetic mutations—an epigenetic property—can result in phyletic trends, parallelisms and convergences.



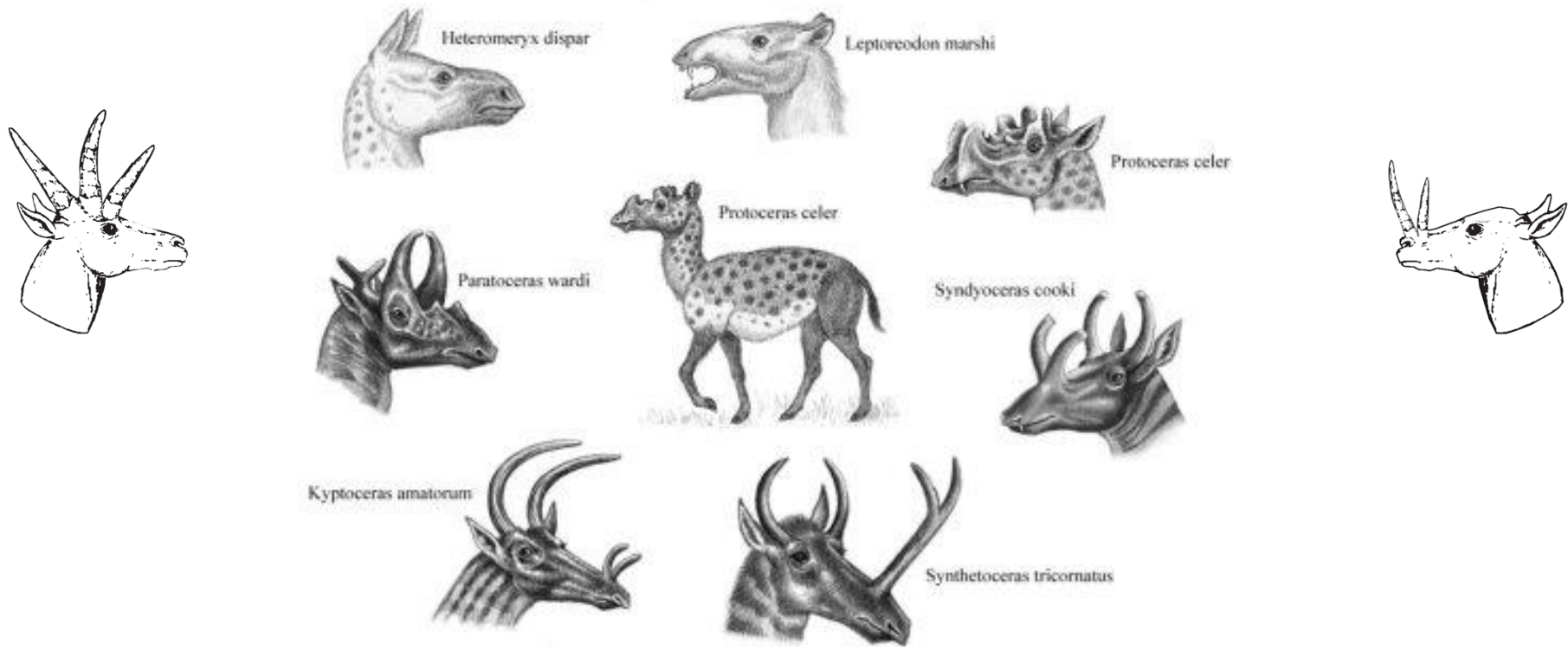
# Empty areas of morphospace – Type 4

“Constraint chasms”



# Empty areas of morphospace – Type 4

“Constraint chasms”

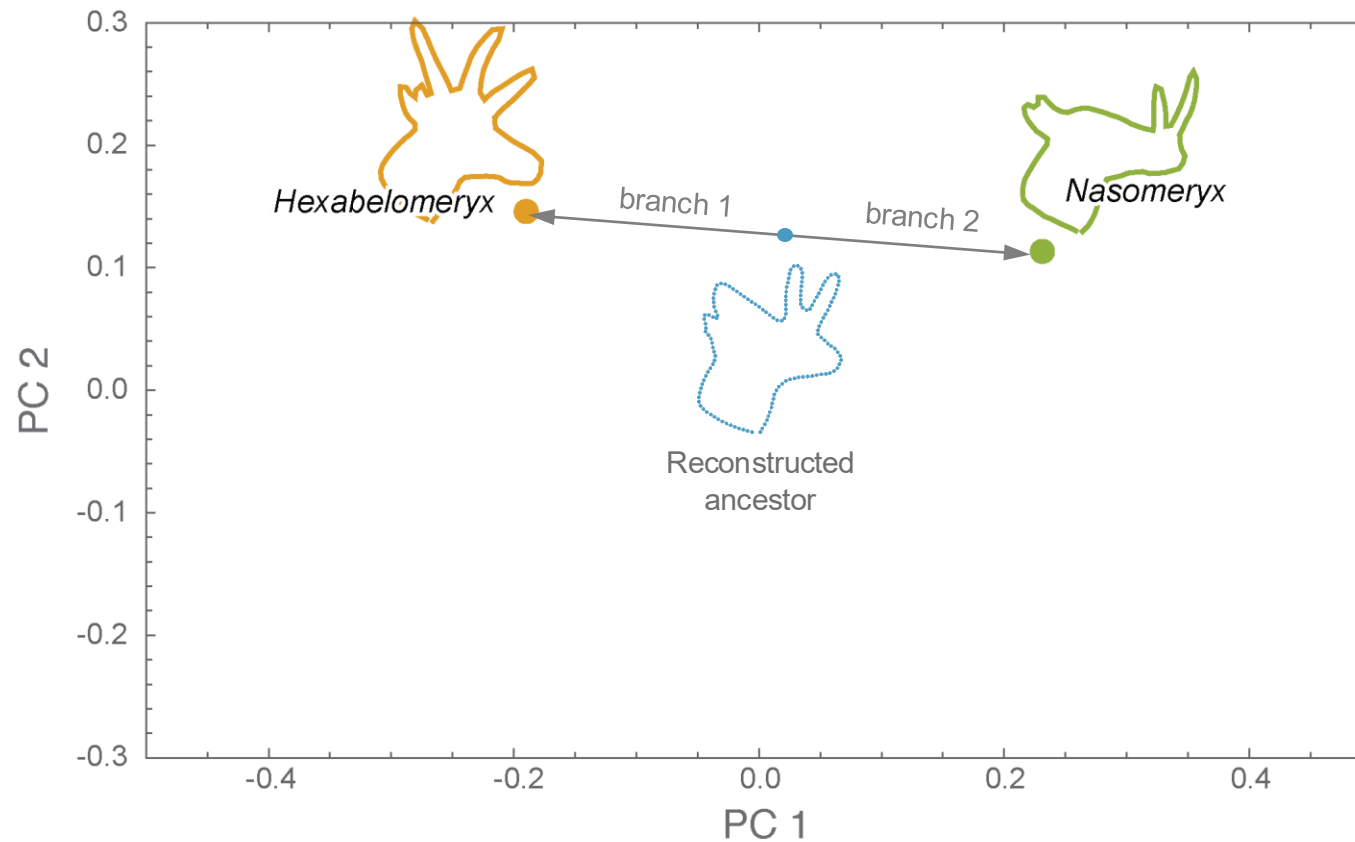


ferrebeekeeper



# Empty areas of morphospace – Type 4

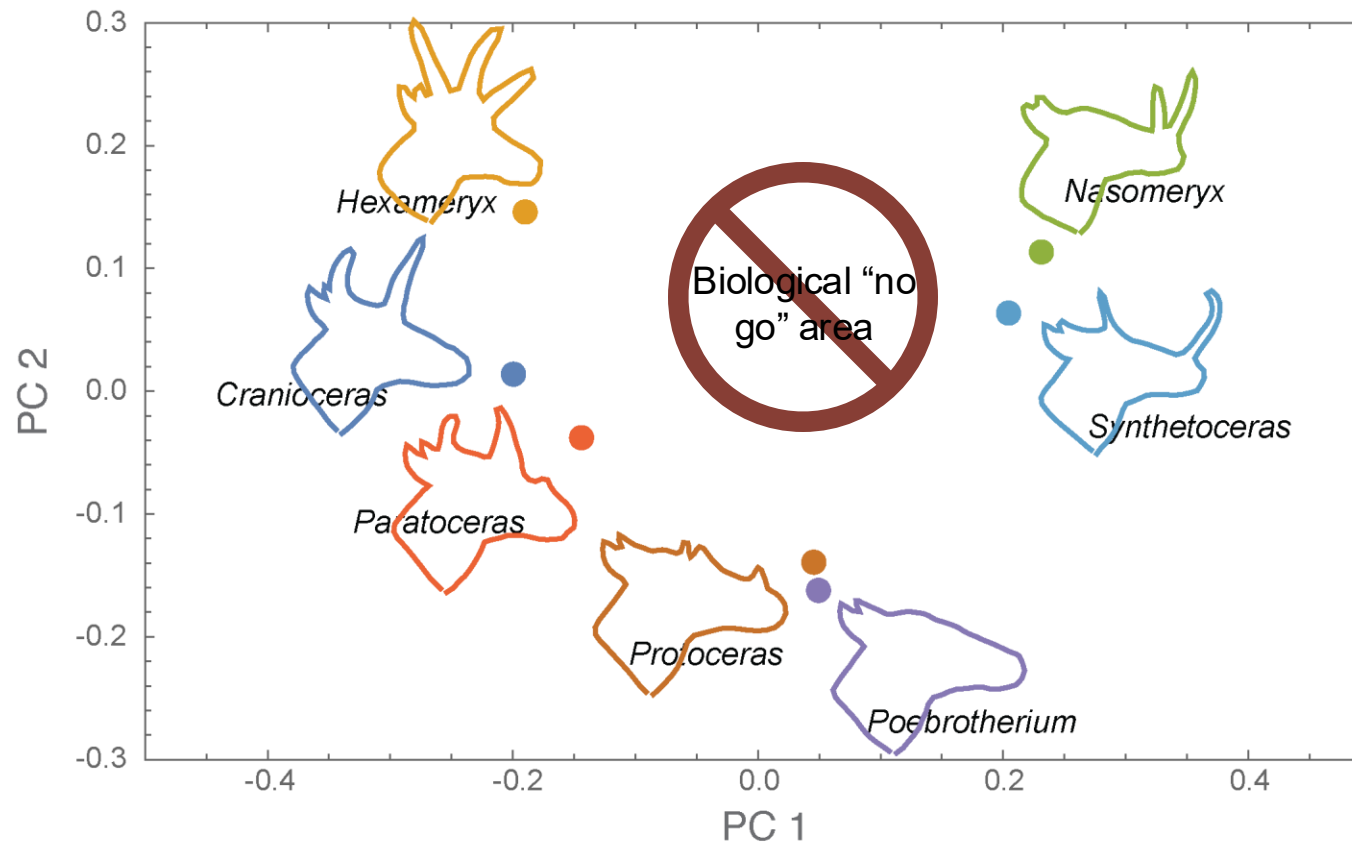
“Constraint chasms”





# Empty areas of morphospace – Type 4

“Constraint chasms”



# Example Type 4: mammal teeth

Isaac Salazar-Ciudad

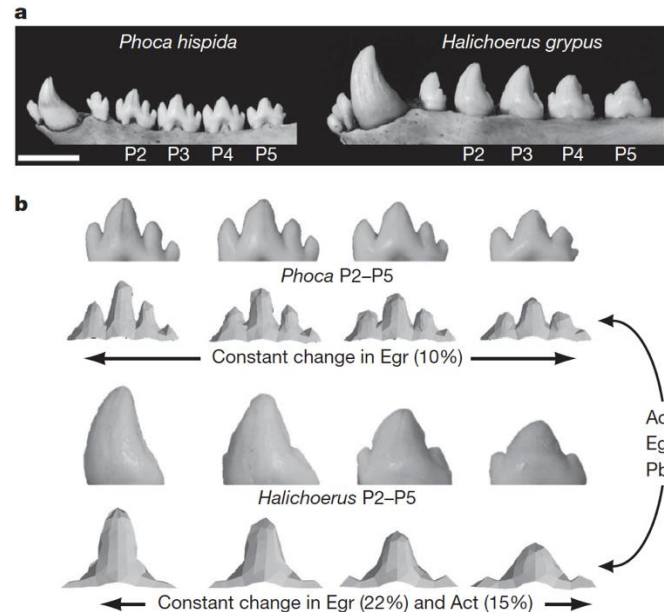


Figure 4 | Serial tooth-to-tooth variation implicates a cellular parameter.

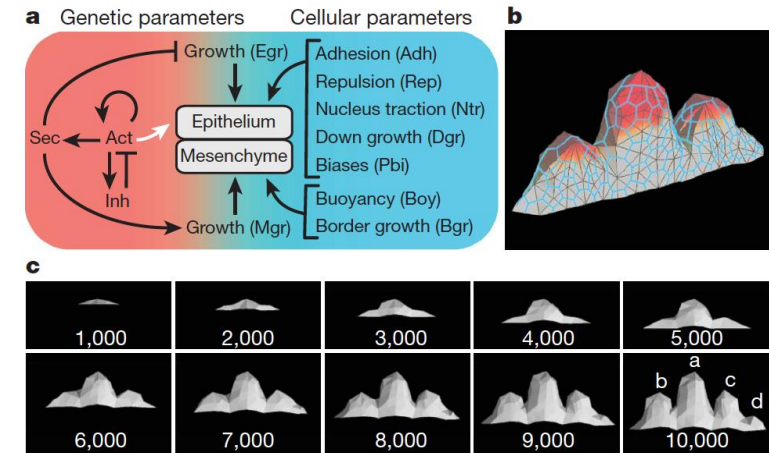
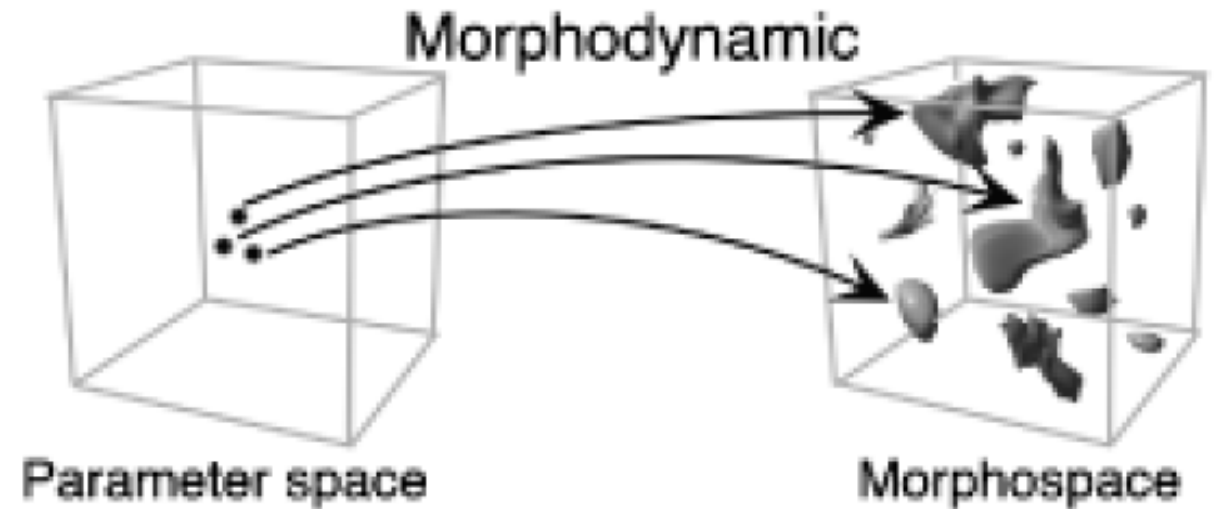


Figure 1 | A model integrating gene networks and tissue mechanics. a, Nine



Salazar-Ciudad I, Jernvall J, 2004, *Evolution and Development*, **6**: 6-16; 2010, *Nature*, **464**: 583–586.



# Empty spaces summary

## Type 1 – ordinary gaps

- everything good

## Type 2 – Kendall curvature

- usually correctable with projection to Euclidean space
- usually negligible for biological data sets (e.g., phylogenetic ones)
- can be an issue with:
  - non-biological objects (e.g. stone tools)
  - simulated shapes
- testable by comparing full and projected Procrustes distances

## Type 3 – Bookstein bends

- fairly common
- can easily be overinterpreted on low PCs
- statistics and model fitting may thwarted on low PCs
- may misguide ancestor reconstruction
- using full morphospace (all PCs) should alleviate statistical issues

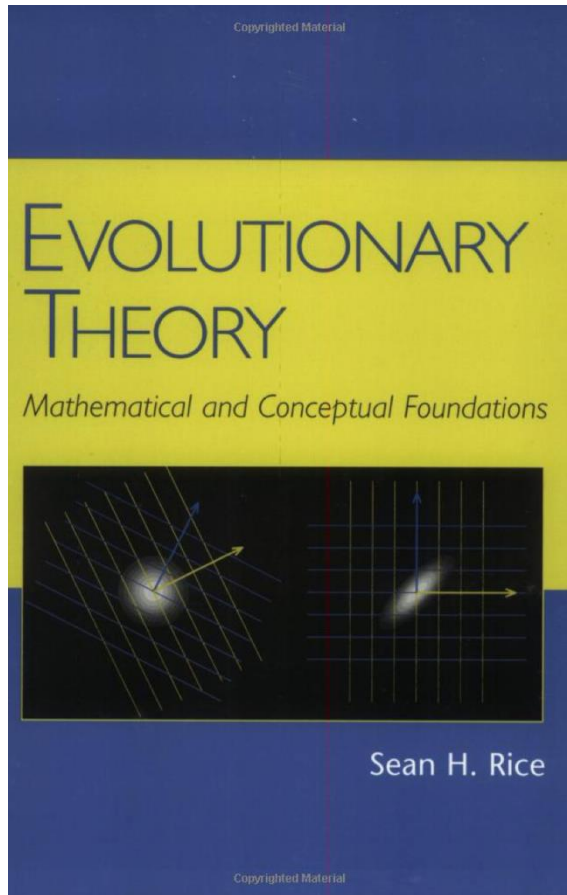
## Type 4 – Constraint chasms

- biological, not mathematical in nature
- caused by non-linear relationship between biological and mathematical paths
- may thwart ancestor reconstruction and model fitting (but not statistics?)

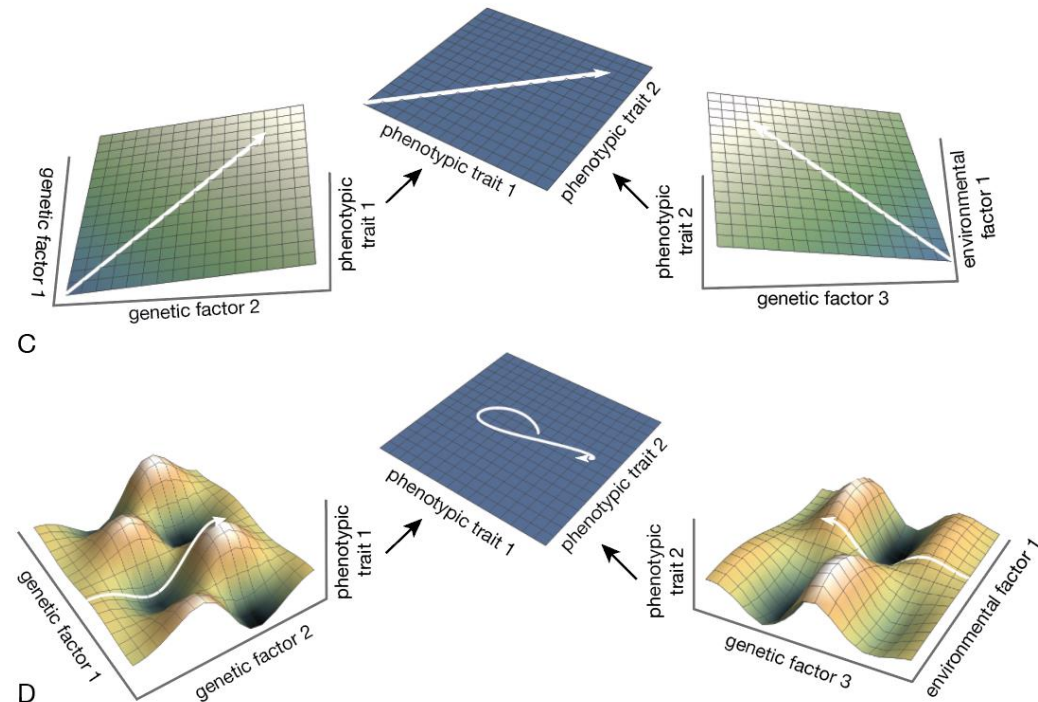


# Phenotypic landscape theory to the rescue?

Rice S, 2004. *Evolutionary Theory*. Sinauer Associates.



Genotype-phenotype map combined with morphometrics could be used to reconstruct biologically realistic trajectories



Polly PD, 2008. *Evolutionary Biology*, 35: 83-96

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(or as otherwise noted)



# Review of morphospace concepts

## The Concept of Morphospaces in Evolutionary and Developmental Biology: Mathematics and Metaphors

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### Abstract

Formal spaces have become commonplace conceptual and computational tools in a large array of scientific disciplines, including both the natural and the social sciences. Morphological spaces (morphospaces) are spaces describing and relating organismal phenotypes. They play a central role in morphometrics, the statistical description of biological forms, but also underlie the notion of adaptive landscapes that drives many theoretical considerations in evolutionary biology. We briefly review the topological and geometrical properties of the most common morphospaces in the biological literature. In contemporary geometric morphometrics, the notion of a morphospace is based on the Euclidean tangent space to Kendall's shape space, which is a Riemannian manifold. Many more classical morphospaces, such as Raup's space of coiled shells, lack these metric properties, e.g., due to incommensurably scaled variables, so that these morphospaces typically are affine vector spaces. Other notions of a morphospace, like Thomas and Reif's (1993) skeleton space, may not give rise to a quantitative measure of similarity at all. Such spaces can often be characterized in terms of topological or pretopological spaces.

The typical language of theoretical and evolutionary biology, comprising statements about the "distance" among phenotypes in an according space or about different "directions" of evolution, is not warranted for all types of morphospaces. Graphical visualizations of morphospaces or adaptive landscapes may tempt the reader to apply "Euclidean intuitions" to a morphospace, whatever its actual topology might be. We discuss the limits of metaphors such as the developmental hourglass and adaptive landscapes that ensue from the geometric properties of the underlying morphospace.

### Keywords

adaptive landscapes, affine space, developmental hourglass, morphometrics, phenetic space, sequence space, shape space, skeleton space, theoretical morphology, topology





# Thinking forward...

Are there better ways for evaluating “empty” spaces?

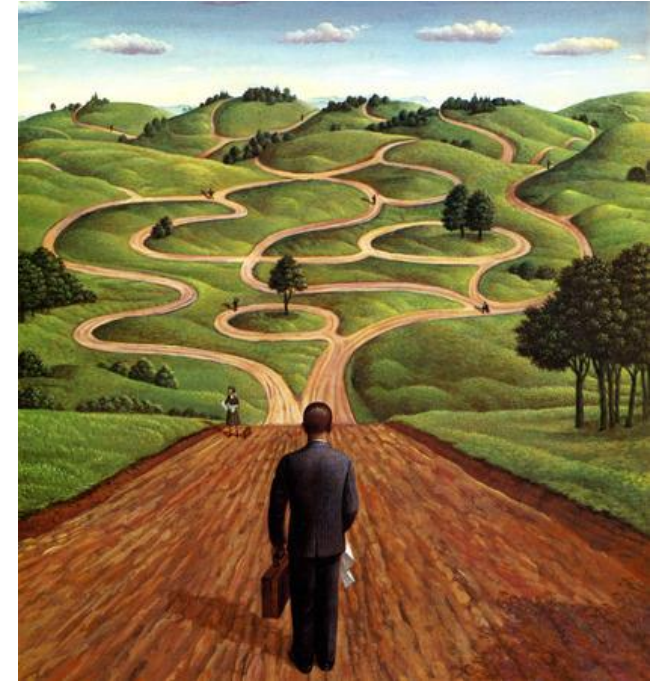
- tests for “Bookstein bends” vs. “Constraint chasms”

Are there better ways for managing high-dimensionality?

- Trade-off between analysis analysis in full vs reduced dimensionality (captures full patterns but over parameterized?)

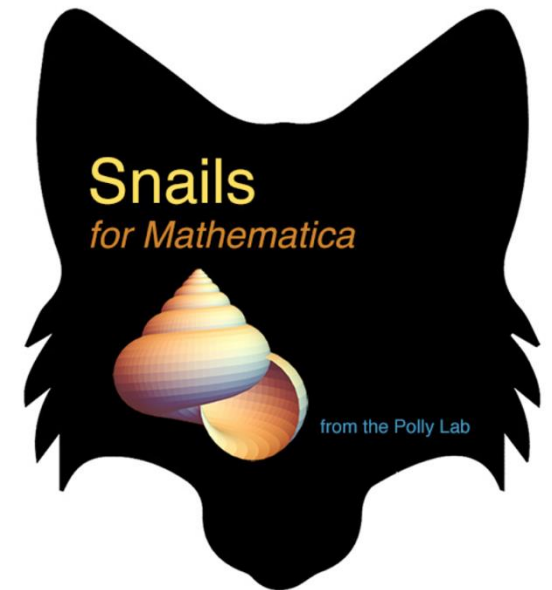
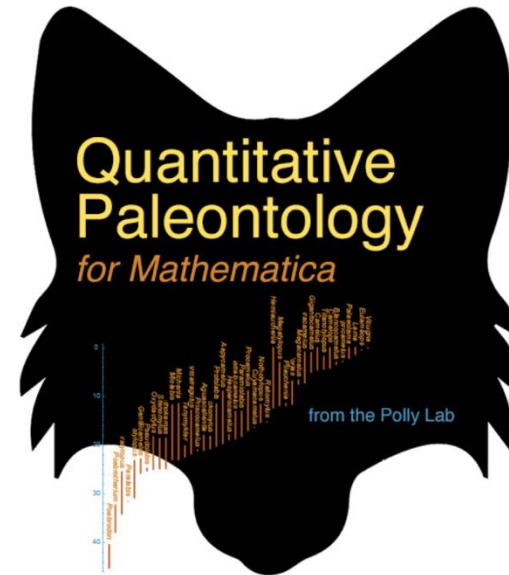
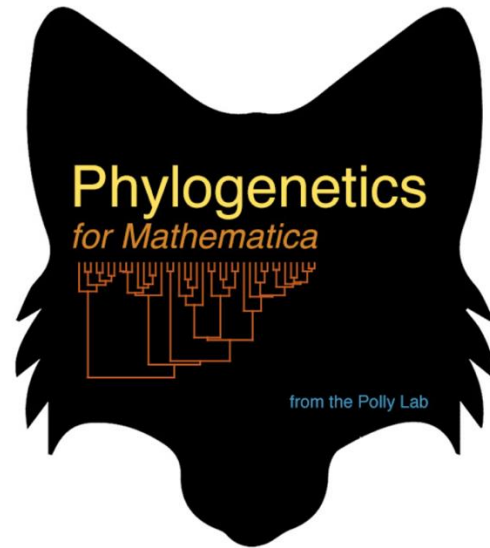
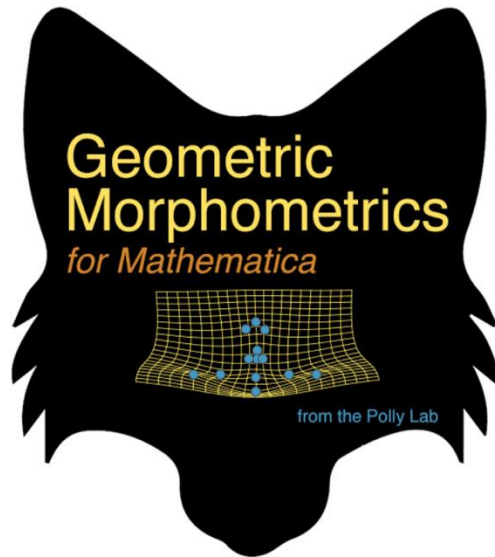
Can easy-to-use tools be developed?

- tests for effects of data pattern on evolutionary models?



# Software for Mathematica©

<https://github.com/pdpolly>







Jason Head, Cambridge



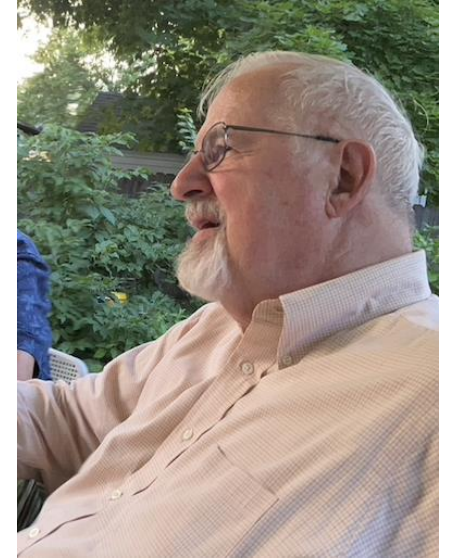
Andrea Cardini  
Modena



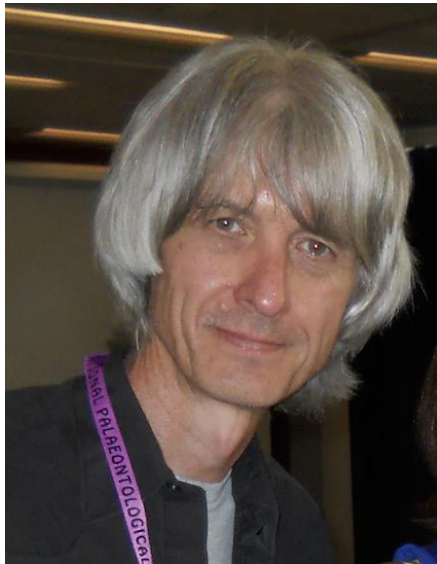
Radha Caumul  
Burty David School, Mauritius



Joe Felsenstein  
University of Washington



Philip Gingerich  
U Michigan



Norman MacLeod, Nanjing U



Michelle Lawing  
Texas A&M



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Arizona State University



Steve Le Comber  
Queen Mary, U London  
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Aida Gómez Robles  
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